



University of Pittsburgh

*Department of Industrial Engineering*

1048 Benedum Hall  
Pittsburgh, PA 15261  
412-624-9830  
Fax: 412-624-9831

## **Variable Expansion and Hybrid Optimization Techniques for Solving Bi-Level Problems**

**Cole Smith**

Department of Industrial & Systems Engineering  
University of Florida

### **ABSTRACT**

Bi-level optimization algorithms such as Benders decomposition have been developed for use in a variety of settings, particularly those involving multiple decision-makers and/or stochasticity. This talk will describe two investigations in which standard bi-level approaches are not practically useful, and where methodological innovation is necessary to solve the problems within reasonable computational limits. First, we consider the creation of additional linking master problem variables that represent nonlinear relationships among the original variables, and show that cutting planes generated in this expanded variable space can be substantially tighter than those in the original variable space. In particular, for both applications discussed in this talk, we show that whereas an exponential number of non-dominated cutting planes can be generated at some stage of a bi-level algorithm in the original variable space, a single inequality in the (polynomially) expanded variable space can imply all of these exponentially many cutting planes.

Using this concept, we demonstrate a substantial computational time reduction in solving a three-stage competitive product introduction problem, and modest computational improvements in solving a stochastic telecommunication network design problem. For the latter problem, we analyze reasons why even the improved algorithm fails to converge quickly, and address these problems by prescribing a hybrid integer programming/constraint programming algorithm that substantially outperforms pure mathematical programming approaches. The talk will conclude by providing general guidelines on when each of these new approaches should be considered when solving difficult bi-level optimization problems.