Stability and Performance of Intersecting Aircraft Flows Under Decentralized Conflict Avoidance Rules

Zhi-Hong Mao, Eric Feron, and Karl Bilimoria

***Abstract*—This paper considers the problem of two intersecting aircraft flows under decentralized conflict resolution rules. Considering aircraft flowing through a fixed control volume, new air traffic control models and scenarios are defined that enable the study of long-term aircraft flow stability. For a class of two intersecting aircraft flows, this paper considers conflict scenarios involving arbitrary encounter angles. It is shown that aircraft flow stability, defined both in terms of safety and performance, is preserved under the decentralized conflict resolution algorithm considered in this paper. It is shown that the lateral deviations experienced by aircraft in each flow are bounded.**

I. INTRODUCTION

THE air transportation system is currently the object of intensive research, following the sustained growth of past and forecasted air traffic. The current enroute air traffic control system consists of the following elements:

- A geographical network whose nodes are navigation beacons (VHF Omni-directional Range (VOR) and Distance Measuring Equipment systems (DME)), and whose links are air routes. The aircraft are allowed to fly only along these routes (with some exceptions). Flying on segments connecting two navigation beacons makes the problem of aircraft navigation and automated guidance particularly easy.
- Approximately 1500 enroute air traffic controllers who regulate the aircraft flow across this network and make sure no hazardous situation develops, whereby two aircraft might get too close to each other (aircraft conflicts). The network structure of the aircraft routing system allows the controllers to get *a priori* information on aircraft conflict geometries and their location during nominal operations: Conflicts are usually located at the nodes of the network. Knowing potential conflict locations *a priori* enables the decomposition of the airspace into *sectors*, managed by individual air traffic controllers, and whose boundaries are located away from the network nodes and therefore away from the most common conflict locations.

Many decades of working experience have demonstrated that this network-based architecture is safe. However, it suffers from strong perceived drawbacks, such as systematic indirect routing between origin and destination, and in general a perceived lack of navigation freedom for the pilots. The advent of a relatively new generation of Global Navigation Satellite Systems (GNSS), in particular GPS, has removed in principle the limitations of the ground-based navigation infrastructure. In particular, it is now very easy to obtain precise aircraft position anywhere over the United States and not only on a pre-determined set of routes (although this idea, also named Area Navigation, has been demonstrated to be feasible for many years [3], using the conventional navigation infrastructure, at the expense of improved on-board computational equipment). As a consequence, concepts of operations such as "Free Flight" [21] have been proposed by airlines and by the Federal Aviation Administration (FAA) to remove the routing constraints imposed by the conventional, fixed-route system. Under Free Flight, each aircraft would be able to optimize its trajectory according to several factors such as perceived safety, weather, direct operating costs and coordination with other flights [20]. Some steps toward Free Flight include the National Route Program, whereby qualified aircraft are allowed to fly their preferred route after approval by the air traffic services. However, in order to be implemented on a full scale, the safety of such concepts needs to be proven. In particular, the set of standards over which operational concepts are evaluated has evolved from empirical evaluation decades ago to a sophisticated and very difficult certification process, which makes proving the safety of any new concept of operations a very challenging task. While many years of reliable operation provide evidence of safety for the current air traffic control system, the safety of any new system cannot rely on experience only, as it is very lengthy and expensive to build up. Rather, future air traffic management concepts will draw from appropriate mathematical modeling and engineering analysis techniques. Thus, Free Flight offers a wide array of new challenges and opportunities to the research community.

This paper considers the problem of aircraft flow "stability" under decentralized conflict detection and resolution rules. One of the major issues arising when considering this problem is the proper definition of "stability". In traditional control system terms, the notion of "stability" usually relates to the long term behavior of dynamical systems, which is expected to remain within some acceptable bounds and often to converge toward a specific desired state. For example, individual aircraft stability concepts are tied to the requirement that both aircraft attitude

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Z.-H. Mao is with the Laboratory for Information and Decision Systems, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

E. Feron is with the Laboratory for Information and Decision Systems, International Center for Air Transportation, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge MA 02139 USA (e-mail: feron@mit.edu).

K. Bilimoria is with the NASA Ames Research Center, Moffett Field, CA 94035 USA (e-mail: kbilimoria@mail.arc.nasa.gov).

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and position stay close enough to some reference attitude and position.

Considering problems of air traffic management, the requirement for stability becomes more complex: While aircraft are expected to follow a reference trajectory (as loosely defined as it may be), aircraft are also required to *stay away* from each other to prevent near misses or even airborne collisions. In this context it becomes quite important for the researcher to define appropriate notions of stability. This in turn entails the requirement of appropriately defining the system being worked upon:

Much of the current research focuses on problems involving a finite, usually small number of aircraft. Such a standpoint is useful when designing efficient conflict detection and resolution systems. However, it is not convenient to analyze problems involving aircraft "flows", since interactions occurring within a finite set of aircraft can only have a finite duration.

This paper proposes a complementary view, where many aircraft flow through an otherwise well-defined airspace volume. The motivation behind this standpoint is that, even under Free Flight, many aircraft flow interactions are expected to occur within relatively well-defined parts of the airspace, corresponding to the intersection between one or more optimal routes linking city pairs, for example. This is also similar to an air traffic controllers's current view of the air transportation system, with the volume of airspace being a sector. Note that this framework also appears in [5] and quite recently in [15].

This paper is organized as follows. First, the aircraft flow models are introduced. An appropriate notion of aircraft flow stability is defined, and the decentralized conflict resolution strategy followed by each aircraft is detailed. Second, a proof of interacting aircraft flow stability is provided for the case of two intersecting aircraft flows where aircraft use a decentralized conflict resolution rule. Third, a discussion of the results is presented along with simulations. A comparison is drawn between decentralized and centralized conflict resolution schemes.

II. AIR TRAFFIC AND CONFLICT RESOLUTION MODELS

A. General Considerations

The definition of appropriate models appears to be a significant challenge when considering problems in air transportation [11]. Considering the conflict detection and resolution problem, most authors (including those of this paper) have traditionally concentrated on scenarios involving a *finite* number of aircraft. However, there appears to be a continuing concern about the "domino effect", whereby one conflict resolution maneuver creates new conflicts which in turn need to be solved, etc. In this paper, we will therefore concentrate on a possibly *infinite* number of aircraft flowing through a finite portion of the airspace.

The system under study consists of a given volume of airspace, and a set of aircraft flowing in and out of it, as shown in Fig. 1. The dynamics of the system are determined by the "boundary conditions" that indicate the location, speed and rate at which aircraft enter the given volume of airspace, and by their individual behavior while they fly within this airspace. Clearly, some boundary conditions are unacceptable; *e.g.*,, the case when two aircraft appear into the control region very close to each other and on a head-on collision course.

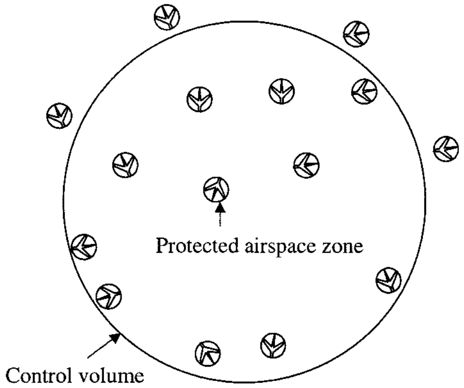


Fig. 1. Aircraft flowing in and out of "control volume".

The aircraft are assumed to be *intelligent*, that is, their pilots actively attempt to maneuver and avoid conflicts at the smallest possible cost.

B. Aircraft Maneuver Models

Although designing and analyzing systems for aircraft conflict detection and resolution needs to account for the three dimensions, this paper will only investigate air traffic evolving in two dimensions (planar conflict resolution): The trajectories of all aircraft are assumed to evolve in the horizontal plane. While vertical maneuvers appear to be most efficient for tactical conflict resolution (such as in the case of TCAS (Traffic Alert and Collision Avoidance System)), horizontal maneuvers might be more appropriate for the "strategic" conflict resolution context considered in this paper, because they induce less passenger discomfort and they do not require flight level changes and thus may not perturb the vertically stratified traffic structure as it exists today in the enroute airspace.

This paper will be concerned with simple aircraft behaviors. In particular, aircraft fly only along straight, level and constant speed trajectories. All aircraft have the same absolute speed. Moreover, we will assume that only one conflict area exists, and that aircraft may perform only one conflict avoidance maneuver [2].

Two models for conflict avoidance will be considered in this paper; Fig. 2 illustrates these conflict resolution models.

- Heading change model: In this model (left picture in Fig. 2), single heading changes (at constant speed) are used to modify aircraft trajectories. Following the approach of Andrews [2], these changes are assumed to occur instantaneously when the aircraft makes a decision. This model will be used for simulation purposes.
- Offset model: This model (right picture in Fig. 2), used for simulation and analysis, consists of modeling aircraft trajectory changes via a single lateral position change, with both speed and heading remaining the same before and after the position change. This model appears to be less realistic; however, it is simpler to use for analysis purposes. In addition, the offset model can be treated as a close approximation of the heading change model, and of the two-stage maneuver model shown in the middle of

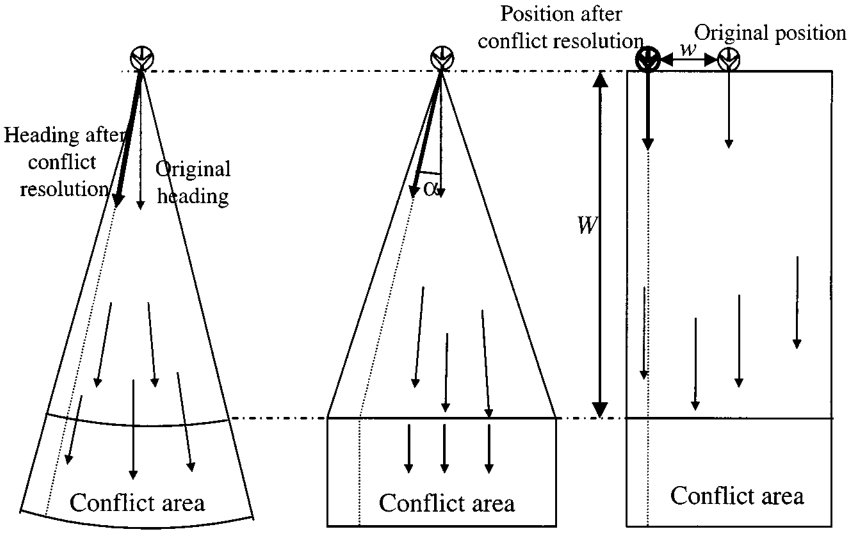


Fig. 2. Heading change model *vs.* offset model. Left: The aircraft maneuver is an immediate heading change. Middle: The aircraft maneuver is a heading change followed by a second heading change. Right: The aircraft maneuver is a relative position change.

Fig. 2. Given the distance to conflict W, the lateral displacement w in the offset model is equivalent to a heading change of amplitude $\alpha = \tan(w/W)^{-1}$ ($\approx w/W$ if W is much greater than w, which is usually the case for strategic conflict resolution.) The longitudinal displacement difference between these two maneuver models is on the order of w^2/W, which will be assumed to be small. Simulations presented hereafter show there is little qualitative difference between the offset and the heading change models for the applications considered in this paper.

The models presented in this paper do not consider recovery maneuvers done subsequently to the conflict resolution maneuver. However, and especially considering the heading change model, one might expect the aircraft to resume their original heading after conflict resolution or even get back to their original track.

C. Aircraft Flow Arrival Geometry

The basic aircraft flow model chosen in this paper is that shown in Fig. 3, originally introduced by Niedringhaus [18]. Two aircraft streams, oriented at a given angle θ relative to each other, feed aircraft into a circular control volume along two tracks that intersect at the center of the conflict area. Prior to maneuvering, we assume all the aircraft to be flying at the same speed, aligned along one of the two tracks. Thus, in the absence of maneuvers, the conflicts all occur at the center of the conflict area. The initial spacing between aircraft in each flow is arbitrary, but no less than a given minimum safe distance d (in practice $d = 5$ nm). Let $A_1, A_2, \ldots, A_i, \ldots$ be the set of aircraft entering the control volume, where aircraft are indexed according to the order they enter the control volume.

D. Conflict Resolution Rules

Several centralized and decentralized conflict resolution rules are available (see for example [7]–[10], [13]–[17], [22], [23]).

A conflict is declared whenever the projected straight paths of any aircraft pair lead them to a miss distance that is strictly less than d. The conflict resolution scheme chosen in this paper follows a decentralized, sequential approach, whereby aircraft solve their own potential conflicts one at a time when they enter the conflict area. To simplify matters, it is assumed that the order in which aircraft perform their resolution maneuver is the same as the order in which they enter the circular conflict area. Hence all conflicts are resolved at the same time-to-conflict, since all aircraft fly at the same speed. An aircraft solving a conflict considers all other aircraft that maneuvered earlier as moving obstacles, but does not account for the aircraft which have not maneuvered yet. Thus each aircraft has knowledge of all aircraft that have already performed a maneuver (or decided that no maneuver was necessary). A reliable implementation of such sequential approaches is described in [1].

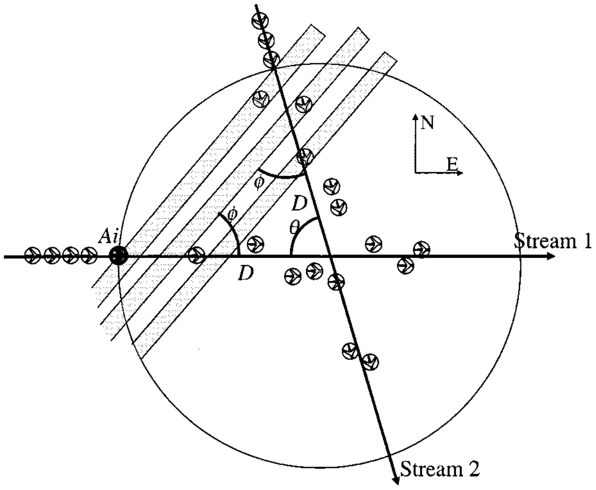


Fig. 3. Aircraft flow configuration for stability analysis. The shaded areas are locations where a conflict will occur.

E. Conflict Resolution Maneuvers

Given those aircraft $A_1, \ldots, A_{i-1}$, which have already performed a resolution maneuver (and must therefore be considered as obstacles), the resolution maneuver for the next aircraft

i scheduled for conflict resolution will be such that (i) no conflict will exist between A_i and $A_1, \ldots, A_{i-1}$ after the resolution and (ii) the amplitude of the conflict avoidance maneuver is as small as possible. For the heading change model, the resolution maneuver minimizes the amplitude of the deviation from the nominal heading. For the offset model, the resolution maneuver minimizes the lateral position change necessary for conflict resolution. It is noted that in both models, the speed remains constant at all times.

F. Definition of Stability

For the purpose of this paper, interacting aircraft flows are defined to be stable if:

1) All conflicts, whether they are direct or created via domino effect, are resolved.
2) The deviation of each aircraft trajectory from its nominal, due to the requirement for conflict resolution, remains bounded.

This definition summarizes the two most important requirements in air traffic control: Safety and efficiency of traffic handling.

III. AIRCRAFT FLOW STABILITY

This section presents the main result of this paper: The system of two intersecting aircraft flows under the flow geometry and conflict resolution rules described above, is stable. More precisely, we ask the following questions: Assuming the system has been running correctly in the past, will it keep running correctly in the future? Is it possible to construct "initial conditions" for the system such that conflicts are unavoidable? We now show that an incoming aircraft can always find a conflict resolution maneuver and proceed with a conflict-free trajectory, and that the magnitude of the conflict resolution maneuver is bounded. We first establish a general result on the amplitude of the deviations undertaken by aircraft. Subsequent simulations will show these estimates are not conservative.

A. Geometrical Problem Formulation

Referring back to Fig. 3, we denote the eastbound aircraft flow as stream 1 and the other aircraft flow as stream 2.

Let $\mathbf{v}_1$ be the velocity vector of aircraft in stream 1 and $\mathbf{v}_2$ be the velocity vector of aircraft in stream 2. From Section II-B, we have $||\mathbf{v}_1|| = ||\mathbf{v}_2||$, where $||\cdot||$ denotes the Euclidean norm. Let $\theta \in (0, \pi)$ be the encounter angle between the two aircraft streams. Let ϕ be the angle between the relative velocity vector $\mathbf{v}_1 - \mathbf{v}_2$ and $\mathbf{v}_1$. ϕ is also the angle between the relative velocity vector $\mathbf{v}_2 - \mathbf{v}_1$ and $\mathbf{v}_2$, and $\phi = \pi/2 - \theta/2$.

Without loss of generality, we may assume that the next aircraft A_i to perform a resolution maneuver is from stream 1 (eastbound), as represented in bold in Fig. 3. By our definition of the aircraft flow and allowable maneuvers, A_i does not conflict with any other aircraft in stream 1. In addition, each aircraft in stream 1 ahead of A_i is within the control volume and has already performed a resolution maneuver. Each aircraft in stream 2 inside the control volume, having already maneuvered, projects a linear, slab-shaped "shadow" of width d, centered around the aircraft and aligned with the relative velocity vector $\mathbf{v}_2 - \mathbf{v}_1$. This shadow is therefore oriented at the angle ϕ relative to the velocity vector of aircraft A_i. Consider a circle C_i of radius $d/2$ centered around A_i (note that the size of the aircraft drawn in all figures is considerably exaggerated). For aircraft A_i to avoid any conflict, it must maneuver so that the circle C_i does not intersect any of the shaded areas. Failure to do so means that a conflict will occur.

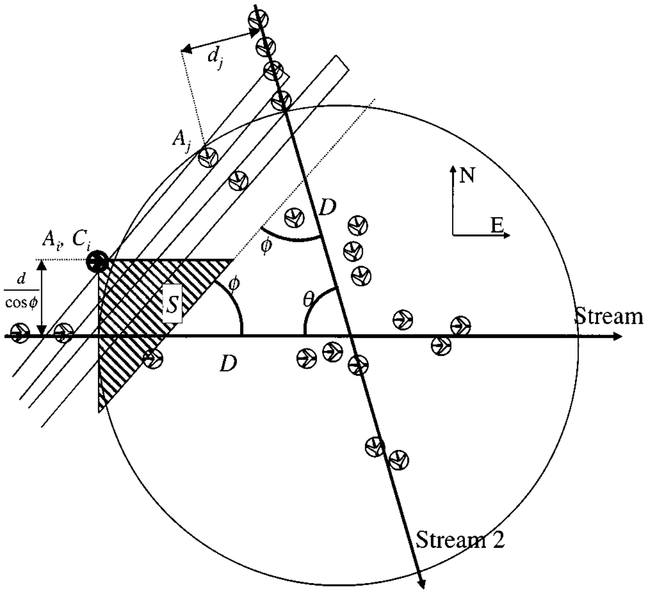


Fig. 4. Bounded conflict resolution maneuvers results from aircraft attempting to minimize their deviation from nominal trajectory.

B. Existence of Bounded Conflict Resolution Maneuvers for Two Intersecting Aircraft Flows

The following theorem states that the system under study is stable.

Theorem: The lateral deviation of aircraft in stream 1 or 2 is bounded above by

$$d_{\max} = \frac{d}{\cos\phi} = \frac{d}{\sin\frac{\theta}{2}}. \tag{1}$$

Proof: Consider any aircraft A_i just entering the control volume and about to make a resolution maneuver. Without loss of generality, one may assume the aircraft A_i belongs to the first, eastbound aircraft stream, as shown in Fig. 4.

We begin with the hypothesis that there exists *no* conflict resolution maneuver with amplitude less than or equal to $d_{\max}$, and then make the following argument.

First, according to the hypothesis, the aircraft A_i should not be able to find a maneuver of amplitude less than or equal to $d_{\max}$ in such a way that the circle C_i, centered around A_i, can be covered by one of the "shadows" of a preceding eastbound aircraft (in stream 1). Otherwise, aircraft A_i could hide itself behind one of these "shadows" and thus succeed in finding a lateral maneuver that results in a conflict-free trajectory, which contradicts our hypothesis. In other words, no eastbound aircraft should stand in the striped triangular area denoted S in Fig. 4 at the time A_i makes a resolution maneuver.

Second, the hypothesis that no lateral maneuver with amplitude less than or equal to $d_{\max}$ exists for which the trajectory of aircraft A_i is conflict-free implies that for all possible lateral deviations of A_i with amplitude less than or equal to $d_{\max}$, the circle C_i intersects the "shadow" projected by at least one aircraft in stream 2, which has already maneuvered. In particular, the interior of the shadow of some aircraft A_j (having performed a maneuver of amplitude d_j) overlaps the interior of C_i whenever A_i makes a lateral displacement to the left with amplitude $d_{\max}$, as shown in Fig. 4. However, because there is no eastbound aircraft in the triangular area S, a right lateral displacement for aircraft A_j such that the shadow projected by A_j is just "tangent" to C_i would have resulted in a conflict-free trajectory for aircraft A_j as well, and this maneuver would have had amplitude strictly smaller than d_j. Since aircraft A_j was supposed to have made an optimal maneuver (minimum amplitude lateral deviation leading to conflict-free trajectory), a contradiction is reached.

Note that condition (1) provides an upper bound on the conflict avoidance maneuver amplitude for both aircraft flows since the situation is symmetric.

Q.E.D.

IV. SIMULATIONS

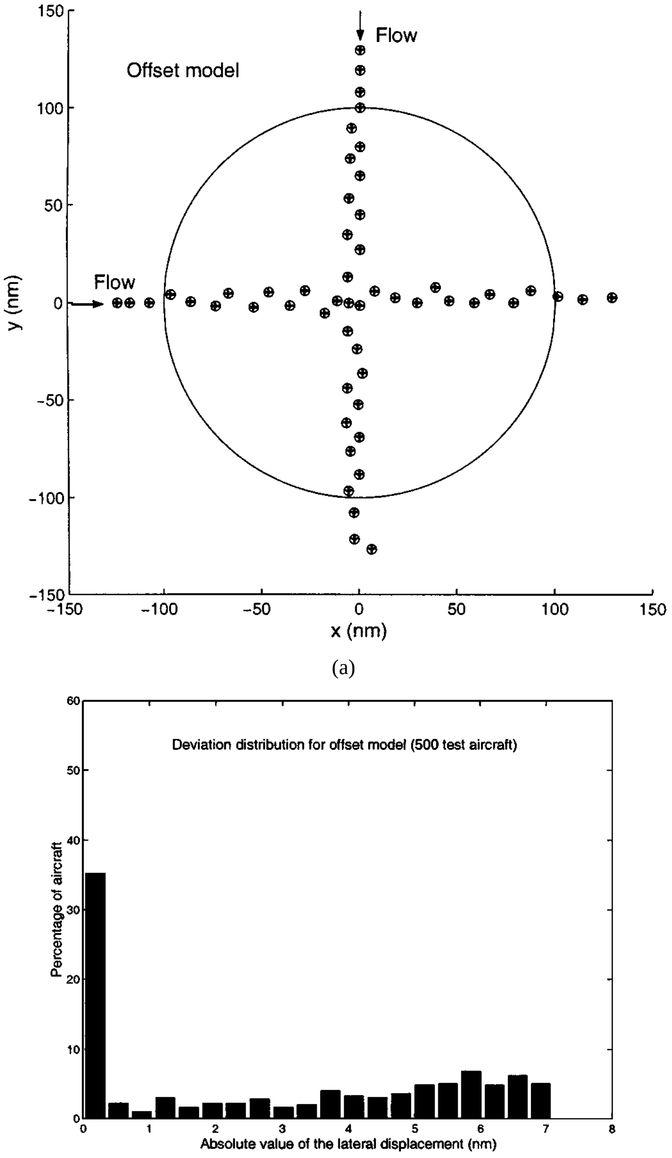
This section presents traffic simulations under the conditions previously described. The goal of the simulations is to do the following.

(a) Numerically verify the upper bound on lateral deviations given in (1) above.
(b) Generate some insight about the structure of the traffic flow after resolution. and
(c) Evaluate the degree of discrepancy between the offset model used for the analysis and a more realistic heading change model.

In the computer simulations presented hereafter, a line search is used to determine conflict resolution maneuvers for both heading change and offset models.

A. Random Arrival Geometry

We first examine the lateral deviations, using the offset maneuver model, of aircraft in two orthogonal ($\theta = 90$ deg) intersecting streams for random arrival patterns. The aircraft in each stream are initially separated by a distance chosen from a uniform distribution over the interval $[5, 15]$ nm. The considered airspace volume (conflict area) is circular with radius 100 nm. A total of 500 aircraft flowing through this airspace have been simulated. Fig. 5(a) gives a snapshot of the traffic flow taken during the conflict resolution process. Also shown in Fig. 5(b) is a histogram of the lateral deviations experienced by the 500 tested aircraft. The largest lateral displacement found in this simulation is 7.1 nm, which turns out to be the upper bound given in (1). The average of the absolute lateral displacements of the tested aircraft is 2.93 nm, and there are 47% of the 500 tested aircraft deviating from the nominal path with a lateral deviation larger than 3.5 nm, half of the upper bound. It is also worth noting that the lateral deviation experienced by aircraft is small compared with typical "distance to conflicts".



(a)

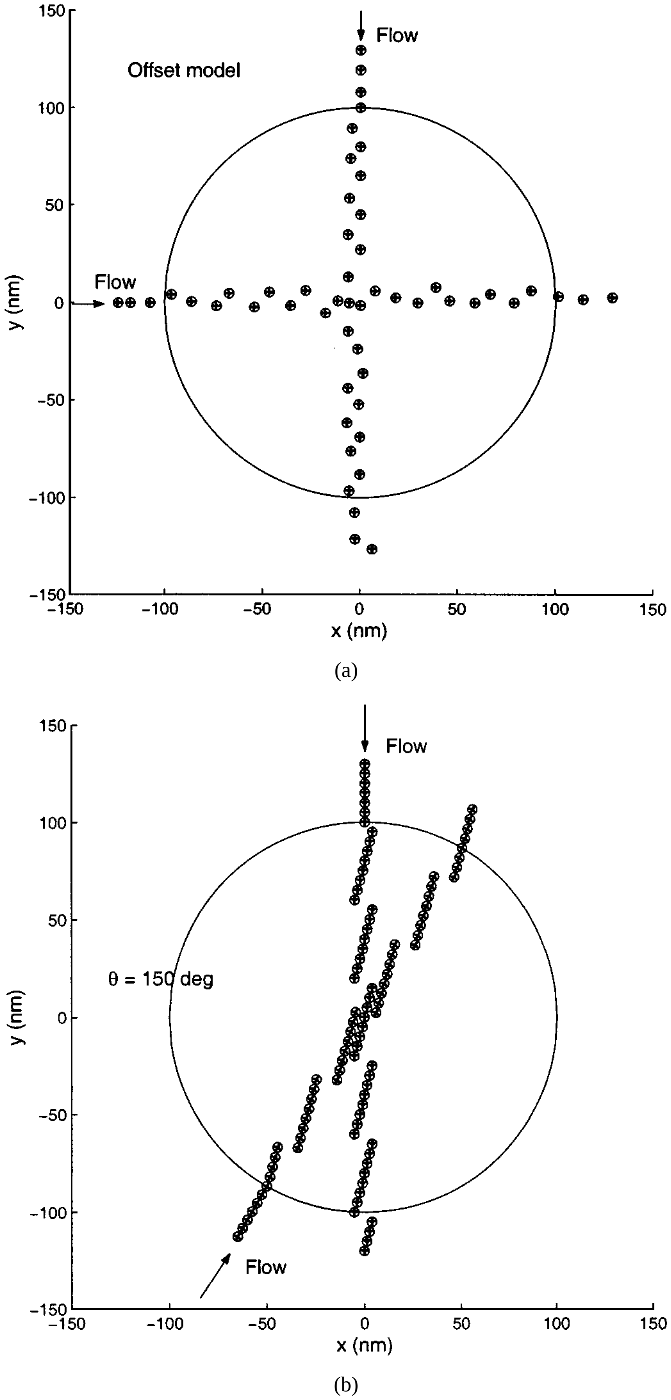
(b)

Fig. 5. Test case for random arrival geometry using the offset model. The separation distance is subject to a uniform distribution on the interval $[5, 15]$ nm.

B. Uniform Arrival Geometry

The following two simulations, using the offset maneuver model, consider the conflict resolution for two streams of aircraft with fixed initial separation distance. Although this uniform aircraft arrival geometry is unrealistic, it may help the reader get some intuition (in addition to the stability analysis) about how the proposed avoidance rules can successfully handle the conflict resolution for aircraft streams. The radius of the conflict area is again 100 nm.

In the first simulation, the two aircraft streams are orthogonal to each other ($\theta = 90$ deg). We chose the initial separation distance of aircraft to be 5 nm, which implies that in this case the aircraft are "packed" in the most compact way before they flow into the conflict area. Moreover, the first eastbound aircraft and the first southbound aircraft enter the control volume at the same time. (When two aircraft enter the control volume exactly at the same time, the southbound aircraft is indexed first and is



(a)

(b)

Fig. 6. Test cases for two aircraft flows with different encounter angles.

given priority.) Fig. 6(a) presents a snapshot of the flows for this case during the conflict resolution process.

The second simulation, shown in Fig. 6(b), was run under the same conditions as those of the first simulation except that the encounter angle of the two aircraft flows was chosen to be $\theta = 150$ deg.

The largest lateral displacement of the aircraft in the above two simulations are 7.1 and 5.2 nm, respectively, which are the same as the upper bound given by (1).

From the above two examples we also observe that uniform aircraft arrival flows generate periodic, conflict-free aircraft flow patterns under the decentralized, sequential conflict avoidance rule. Furthermore, the conflict avoidance rule groups the aircraft in "platoons", and each platoon is formed in such a manner that the aircraft in a platoon share the same "shadow". Note the number of aircraft in each platoon is different depending upon the encounter angle θ. Intuitively, this kind of platooning is very efficient for conflict resolution involving two aircraft flows. The platooning results in a "shearing" motion when two platoons (from the two aircraft streams) meet at the center of the conflict area. Interesting enough, platooning has been proposed as a viable, although heuristic option in many intelligent, hierarchical transportation systems [24], [6].

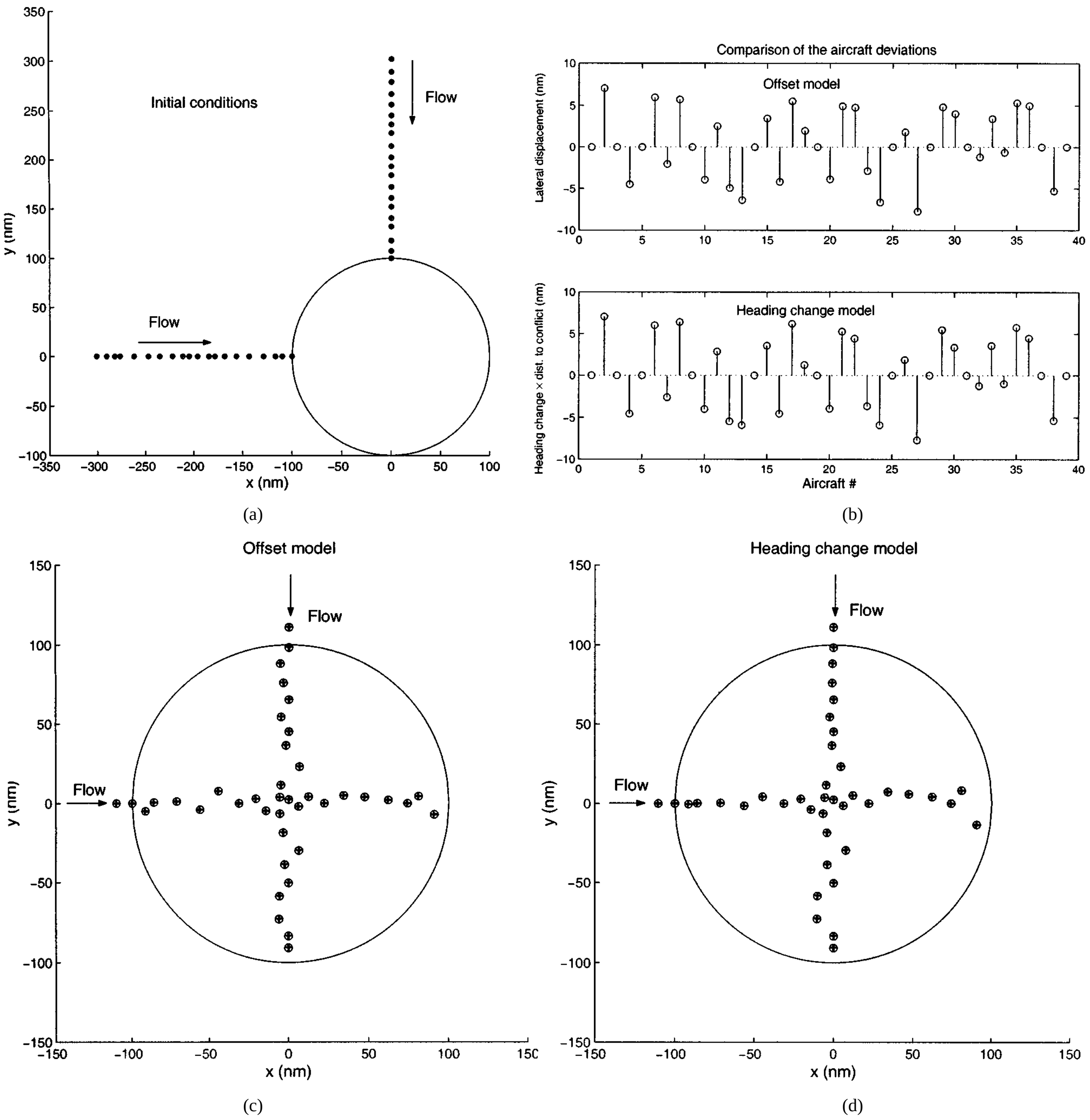
C. Offset vsersus Heading Change Model

To evaluate the impact of the modeling discrepancies between the offset model and the heading change model, a limited experiment was performed: Considering the case of two orthogonal aircraft flows (one eastbound flow, one southbound flow), the same sequence of 40 incoming aircraft $A_1, \ldots, A_{40}$ (with random arrival spacing) was simulated using the offset model and the heading change model. In this simulation, the radius of the conflict area is $D = 100$ nm. Under the offset model, the lateral deviations, denoted $d_1, \ldots, d_{40}$ were recorded. A snapshot of the traffic is shown in Fig. 7(a). This deviation is assumed to be positive if the maneuver is to the right, negative if it is to the left. Under the heading angle change model, the angular deviations are denoted $\psi_1, \ldots, \psi_{40}$. Likewise, these angles are positive if the deviation is to the right, negative otherwise. In Fig. 7(b), the deviations $d_1, \ldots, d_{40}$ and $D\psi_1, \ldots, D\psi_{40}$ were plotted. Indeed, the latter quantities represent the lateral deviation experienced in the center of the conflict area by aircraft under the heading change model. If the effects of both models are similar, these quantities should be similar. As seen in Fig. 7(b), there is no significant qualitative or quantitative difference between the resulting aircraft deviation amplitudes, thus suggesting the offset model used for the analysis is a valid approximation to the more realistic heading model, for the applications considered in this paper.

V. COMPARISON WITH CENTRALIZED CONFLICT RESOLUTION STRATEGIES

The offset model makes it possible and fairly easy to compare the solutions provided by decentralized, sequential conflict resolution strategies with centralized, optimal resolution strategies that may be obtained for a large but finite set of aircraft. The goal of this section is to evaluate the degree of "inefficiency" or performance degradation of the decentralized conflict resolution scheme discussed in the earlier paragraphs relative to the benchmark centralized solution.

Considering two aircraft streams as introduced previously, we assume that the number of aircraft is now finite (the two aircraft streams are truncated) and that all aircraft maneuver simultaneously using the offset model. Let $A_1, \ldots, A_n$ be the set of aircraft under consideration. The centralized optimization is that of minimizing the maximum absolute value of the lateral deviation experienced by any aircraft, subject to the constraint that all conflicts be solved. Such an optimization problem may be easily written as a mixed integer programming problem.



(a) (b)

(c) (d)

Fig. 7. Offset model vs. heading change model.

Referring to Fig. 8, aircraft A_i and A_j (assumed not to belong to the same flow and to travel at the same constant speed) will not be in conflict if and only if the circle centered at A_i with a radius of $d/2$ does not intersect the "shadow" projected by aircraft A_j. Let $\mathbf{P}_{i0} = (x_{i0}, y_{i0})$ be the original position vector of aircraft A_i, and $\mathbf{P}_i = (x_i, y_i)$ be the position vector of A_i after resolution. Without loss of generality, we assume that the center of the conflict area is located at the origin (0,0). Since the aircraft are assumed to resolve conflicts via lateral position changes only, $\mathbf{P}_i$ should satisfy the equality constraint

$$(\mathbf{P}_i - \mathbf{P}_{i0})^T \mathbf{P}_{i0} = (x_i - x_{i0})x_{i0} + (y_i - y_{i0})y_{i0} = 0. \quad (2)$$

Aircraft A_i and A_j are not in conflict if and only if

$$\begin{aligned} x_i \tan\phi + y_i &\geq x_j \tan\phi + y_j + \frac{d}{\cos\phi} \\ x_i \tan\phi + y_i &\leq x_j \tan\phi + y_j - \frac{d}{\cos\phi}, \end{aligned} \quad (3)$$

where ϕ is the angle between the relative velocity vector $\mathbf{v}_i - \mathbf{v}_j$ and $\mathbf{v}_i$. Note that both (2) and (3) are linear in the decision variables x_i and $y_i, i, j = 1, \ldots, n$ and A_i and A_j do not belong to the same flow.

The lateral position deviation of aircraft A_i is given by

$$\frac{|x_i y_{i0} - x_{i0} y_i|}{\sqrt{x_{i0}^2 + y_{i0}^2}}. \quad (4)$$

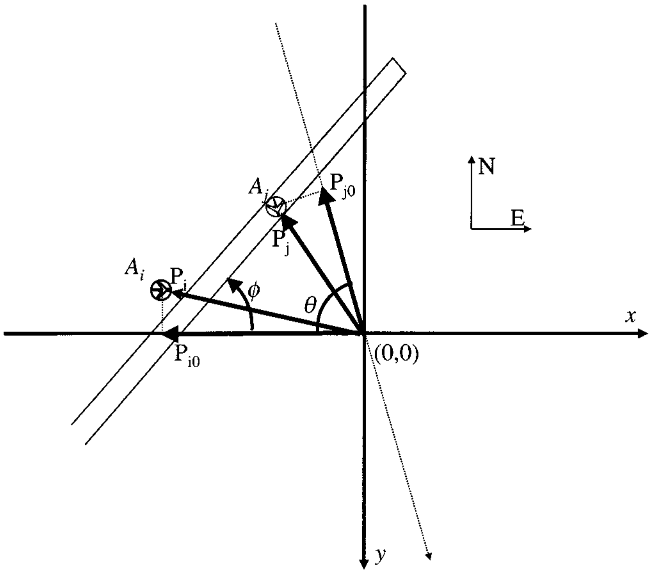


Fig. 8. Conflict avoidance constraints.

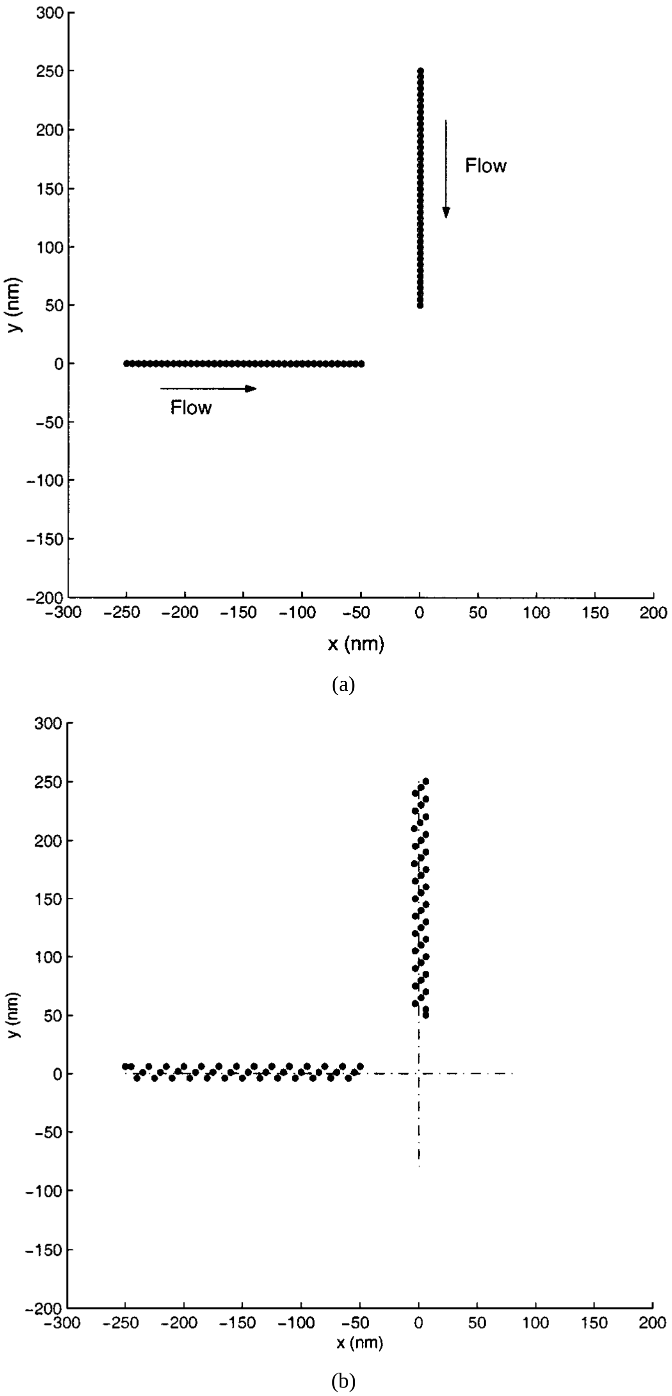
The centralized conflict resolution problem may therefore be written as

$$\begin{aligned} &\text{Minimize} \quad \max_{i=1,\ldots,n} \frac{|x_i y_{i0} - x_{i0} y_i|}{\sqrt{x_{i0}^2 + y_{i0}^2}} \\ &\text{subject to} \quad (2) \text{ and } (3), \end{aligned} \tag{5}$$

where the decision variables are x_i and y_i, $i = 1, \ldots, n$. This problem may be reformulated as the mixed integer program

$$\begin{aligned} \text{Minimize} \quad & \gamma \\ \text{subject to} \quad & -\gamma \leq \frac{x_i y_{i0} - x_{i0} y_i}{\sqrt{x_{i0}^2 + y_{i0}^2}} \leq \gamma, \\ & (x_i - x_{i0}) x_{i0} + (y_i - y_{i0}) y_{i0} = 0, \\ & M t_{ij} + x_i \tan\phi + y_i \geq x_j \tan\phi + y_j \\ & \quad + \frac{d}{\cos\phi}, \\ & M(t_{ij} - 1) + x_i \tan\phi + y_i \leq x_j \tan\phi \\ & \quad + y_j - \frac{d}{\cos\phi}, \\ & i = 1, \ldots, n, \quad j = 1, \ldots, n, \quad i \neq j, \end{aligned} \tag{6}$$

where the continuous decision variables are x_i and y_i for aircraft A_i, t_{ij} are binary decision variables (they are restricted to belong to the set $\{0, 1\}$), and M is a large constant. We refer the reader to books on mixed integer programming such as [12] for details about the meaning of the variables t_{ij} and the constant M. Intuitively however, the effect of the variables t_{ij} is to "turn on" or "turn off" either one of the avoidance constraints, depending upon the value taken by t_{ij}, thus providing a set of constraints equivalent to (3). This problem may be solved efficiently by using powerful mixed integer programming optimization software such as CPLEX [4].



(a)

(b)

Fig. 9. Test case for uniform arrival geometry using centralized conflict resolution. The initial separation distance is 5 nm. Upper: before resolution. Lower: after resolution.

Fig. 9 shows the conflict resolution for two orthogonal streams of aircraft, with a total of 82 aircraft. In this example, the initial separation distance between aircraft is 5 nm and the aircraft are initially configured as shown in Fig. 9(a), which is the same as the initial conditions for Fig. 6 (left picture). Compared with Fig. 6 (left picture), Fig. 9(b) reveals a slightly more compact conflict resolution structure: The largest lateral displacement experienced by the aircraft in Fig. 9 is 6.1 nm, which is smaller than the 7.1-nm maximum displacement of aircraft in the decentralized resolution scheme.

VI. CONCLUSION

In this paper, we have considered the problem of demonstrating stability of two interacting aircraft flows. We have shown for general aircraft encounter scenarios that these interacting flows remain stable under decentralized, sequential conflict avoidance, and derived upper bounds on the deviation experienced by the aircraft. Simulations indicate the obtained bounds are consistent with the observed aircraft displacements. A simulation of centralized conflict resolution was also conducted. Based on a limited comparison of simulation results, it was observed that decentralized sequential conflict resolution yields the same traffic flow patterns (platoons) as the centralized conflict resolution, albeit some performance degradation (as measured by trajectory deviations). The models considered in this paper do not consider uncertainties such as aircraft deviations from their original trajectory, pilot latency etc. These points will be considered in further research.

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Zhi-Hong Mao received the dual Bachelor's degrees in automatic control and mathematics in 1995 and the Master of Engineering degree in intelligent control in 1998, all from Tsinghua University, Beijing, China, and the Master of Science degree in aeronautics and astronautics in 2000 from MIT, Cambridge. He is currently working toward the Ph.D. degree in medical engineering at the Harvard-MIT Division of Health Sciences and Technology, MIT.

Eric Feron received the Diplome D'Ingenieur, Diplome D'Etudes Approfondies from Ecole Polytechnique, Ecole Normale Superieure and the Ph.D. degree from Stanford University, respectively.

He has been on the faculty of the Department of Aeronautics and Astronautics at MIT, cambridge, since 1993. His work concentrates on control systems and optimization and their applications to aircraft dynamics, unmanned aerial vehicles, and air transportation.

Dr. Feron is a past Associate Editor for the IEEE TRANSACTIONS ON AUTOMATIC CONTROL. He is the recipient of an NSF Research Initiation Award, and an ONR Young Investigator Award. He is an Advisor to the French National Academy of Technologies.

Karl Bilimoria received the Ph.D. degree in aerospace engineering from Virginia Polytechnic Institute and State University, Blacksburg, in 1986.

He then joined the aerospace engineering faculty at Arizona State University, Tempe, where he held the positions of Assistant Professor and Research Scientist. While at Arizona State University, he taught courses and conducted research in the areas of flight dynamics and optimal control. He joined NASA Ames Research Center, Moffett Field, CA, in 1994 as a Research Scientist. During his career at NASA Ames, he has worked in the area of Air Traffic Management under NASA's Advanced Air Transportation Technologies (AATT) program, and has made significant research contributions in the area of conflict detection and resolution.

Dr. Bilimoria is an Associate Fellow of the AIAA and serves on the Editorial Board of the *Journal of Guidance, Control, and Dynamics*.