BioE 2696/ECE 2695 Control Theory in Neuroscience

Solution to HW 3

Problem 1 Solution:

Solve this using the Goldman equation given in lecture. The answers are -90.75 at rest, and 39.36 for the action potential.

What they need to do is add a term to the equation as it was given in class for chloride. The key thing to realize is that the sign on chloride is negative, which means its term in the log must be inverted. The commonest wrong answers will probably be -53.46 at rest, and 40.80 for the action potential, which is what you get if you ignore the sign on chloride.

Problem 2 Solution:

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% BioE 2696 Homework 3 question 2009-01-21

% Create the matrix of spikes for five trials, 1000 ms of data per trial.
frs = repmat(0,5,1000);
probMat = repmat(0, size(frs));
probMat(:,1:400) = 0.1;
probMat(:,401:600) = 0.7; % An event occurs just before 400 ms
probMat(:,601:1000) = 0.5; % It terminates before 600 ms.
randMat = unifrnd(0,1,size(frs));
frs = randMat < probMat;
```

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% Something like the code from here on down is what the students should
% submit.
% Create a Gaussian kernel for smoothing the spike trains
kern = normpdf([1:50],25,10);

% Conv.m doesn't work on matrices, only vectors, so we have to loop over trials
for trialInd = 1:5
    smoothfrs(trialInd,:) = conv(frs(trialInd,:),kern);
end

% remove the padding introduced by convolving
unpadfrs = smoothfrs(:,25:1025);

% Average over the five trials to make the PSTH
psth = mean(unpadfrs,1);

% The students' plots should look something like what this gives you.
% Note that the problem did not stipulate the width of the Gaussian.
% Smoother or noisier than this one is fine. Indeed, experimentalists often
% tune PSTHs by eye by varying sigma.
plot(psth)

% Acceptable answers for the last part (time of stimulus onset and offset)
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% are in the range of 400 and 600 ms A really good answer is 300 or 500 ms
% (accounting for propagation delays).
% If you get something like 425 and 625, then they did not remove the zero-pad
% after they convolved.

Problem 3 Solution:

(a) $T(s) = \dfrac{K/s^2}{1 + KK_v/s + K/s^2} = \dfrac{K}{s^2 + KK_v s + K}$

(b) dc gain $= T(0) = 1$, $\therefore \; \theta_{ss}(t) = 10°$

(c) Critical damping, $\zeta = 1$

$\omega_n^2 = K$; $\omega_n = \sqrt{K}$

$2\zeta\omega_n = 2\omega_n = 2\sqrt{K} = KK_v$; $\therefore \; K_v = 2(K)^{\frac{1}{2}}$

(d) $\tau = 1.5\,s = \dfrac{1}{\zeta\omega_n} = \dfrac{1}{\sqrt{K}}$, $\therefore \; \sqrt{K} = \dfrac{1}{1.5} = 0.667$

$\therefore \; K = 0.444$; $K_v = \dfrac{2}{0.667} = 3$

(f) $T(s) = \dfrac{K}{s^2 + K}$, $\therefore \; \zeta = 0$

Response is an undamped sinusoid, with
$\omega = \sqrt{K} = 0.667$, $\therefore \; T = \dfrac{2\pi}{\omega} = 9.42\,s$