BioE 2696/ECE 2695 Control Theory in Neuroscience

Solution to HW 2

Problem 1 Solution:

(a)

$$F(s) = \frac{5}{s(s+1)(s+2)} = \frac{2.5}{s} + \frac{-5}{s+1} + \frac{2.5}{s+2}$$

$$f(t) = \underline{2.5 - 5e^{-t} + 2.5e^{-2t}}$$

(b)

$$F(s) = \frac{s-30}{s(s^2+4s+29)} = \frac{s-30}{s[(s+2)^2+5^2]} = \frac{-\frac{30}{29}}{s} + \frac{k_2(s+2)+5k_3}{(s+2)^2+5^2}$$

$k_2 = 30/29,\ \ k_3 = 89/145$

$$f(t) = -\frac{30}{29} + \frac{30}{29}e^{-2t}\cos(5t) + \frac{89}{145}e^{-2t}\sin(5t).$$

Problem 2 Solution:

(a)

$$f(t) = 4e^{-2(t-3)}u(t-3)$$

$$df/dt = -8e^{-2(t-3)}u(t-3) + 4e^{-2(t-3)}\Big|_{t=3}\delta(t-3)$$

$$\mathcal{L}[df/dt] = \frac{-8e^{-3s}}{s+2} + 4e^{-3s} = \underline{\frac{4s\,e^{-3s}}{s+2}}$$

(b)

$$\mathcal{L}[df/dt] = sF(s) - f(0) = sF(s) = \underline{\frac{4s\,e^{-3s}}{s+2}}$$

Problem 3 Solution:

(a) Fig.(a) $C = G_1G_2E + G_2HF$

Fig.(b) $C = G_aE + G_aG_bF$

$\therefore G_a = G_1G_2$ and $G_b = \dfrac{G_2H}{G_a} = \dfrac{H}{G_1}$.

(b) Fig. (c) $C = G_cE + G_dF$

$\therefore \underline{G_c = G_1G_2}$ and $\underline{G_d = G_2H}$

Problem 4 Solution:

(a) Fig. (a) $C = G_1G_2E$ $\quad F = G_1HE$

Fig. (b) $C = G_aE$ $\quad F = G_aG_bE$

$\therefore \underline{G_a = G_1G_2}$ $\quad G_b = \dfrac{G_1H}{G_a} = \underline{\dfrac{H}{G_2}}$

(b) Fig. (c) $C = G_cG_dE$ $\quad F = G_cE$

$\therefore \underline{G_c = G_1H}$ $\quad G_d = \dfrac{G_1G_2}{G_c} = \dfrac{G_2}{H}$

Problem 5 Solution:

(a) Linear (by examining the definition—please give the detailed steps by yourself).

(b) It is NOT time-invariant. Consider $u_{\text{new}}(t) = u(t-s)$ for a constant s, and let us check whether $y_{\text{new}}(t)$ equals $y(t-s)$. Note that $y_{\text{new}}(t)=\int_0^{+\infty} e^{-(t-\tau)}u(\tau-s)d\tau$. By introducing $v = \tau - s$, we have

$$y_{\text{new}}(t)=\int_{v=0-s}^{v=+\infty} e^{-(t-v-s)}u(v)dv=\int_{-s}^{+\infty} e^{-(t-v-s)}u(v)dv=\int_{-s}^{+\infty} e^{-(t-\tau-s)}u(\tau)d\tau$$
$$\neq \int_0^{+\infty} e^{-(t-\tau-s)}u(\tau)d\tau = y(t-s).$$

(c) NON-causal, because we need "future" information of $u(\tau)$ for $\tau > t$ to calculate $y(t)$ at the present time t.

(d) NOT memoryless (please verify by yourself).