BioE 2696/ECE 2695 Control Theory in Neuroscience

Solution to HW 1

Problem 1 Solution: The peak value is $\frac{1}{\sqrt{d}}$, the RMS value is 1 and the average-absolute value is $\sqrt{d}$. As $d \to 0$, the peak value $\to \infty$, the RMS value remains fixed at 1, and the average-absolute value $\to 0$.

In particular for very small d, the peak is very large, the RMS value is one, and the average-absolute is very small. In other words, depending on what method you use to measure the 'size' of this signal, you can get answers ranging from 'very small' to 'very large'.

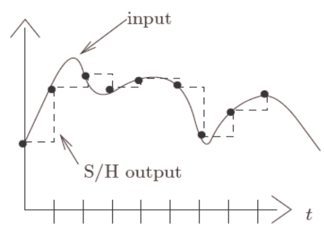
You might find this really confusing, but don't worry. In any practical application there will be some most appropriate method to measure the signal size. Also, for most (but obviously not all) signals, these measures give answers that aren't too far off. (Sinusoids give a good example; see the lecture notes.)

Problem 2 Solution: (a) X[0] = 96, X[31] = 96, X[k] = 0 for 0<k<31.
(b) X[0] = 6, X[k] = 0 for 0<k<6.

Problem 3 Solution:

First let's dicsuss the complicated notation we used. $\lfloor t/h \rfloor$ gives the number of sample times that have occured at time t. Multiplying by h, which yields $h\lfloor t/h \rfloor$ gives the time of the last sample time. The output of a S/H at time t is given by the input signal value at the last sample time, *i.e.*, $u(h\lfloor t/h \rfloor)$.

A sketch of an input signal and its corresponding output is shown below:



The S/H system *is* linear. To check this we must check the homogeneity property and the superposition property. For superposition, suppose you have two signals, u and v, and consider the signal $u + v$. For any t we have

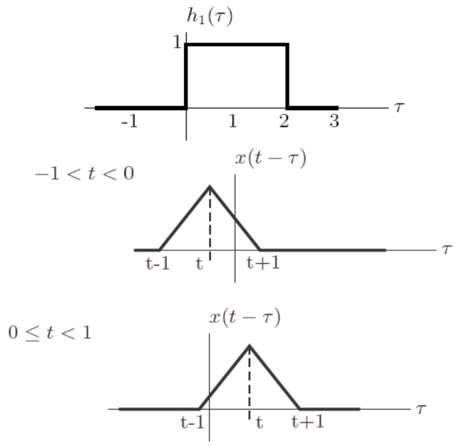
$$(u + v)(h\lfloor t/h \rfloor) = u(h\lfloor t/h \rfloor) + v(h\lfloor t/h \rfloor)$$

which shows that superposition holds. A similar argument shows that a S/H is homogenous.

Problem 4 Solution:

Here, we can break $h(t)$ up into $h(t) = h_1(t) + h_2(t)$ where $h_1(t)$ is the "box" part of $h(t)$ and $h_2(t)$ are the two impulses. Let $y_1(t)$ and $y_2(t)$ denote the result of convolving $x(t)$ with $h_1(t)$ and $h_2(t)$ respectively.

First let us compute $y_1(t)$. To do this, we fix $h_1(t)$ and flip and slide $x(t)$. The following figure illustrates the different regions of overlap.



For the range $-1 < t < 0$, the result of the convolution is the area under the product of the two signals which is given by:

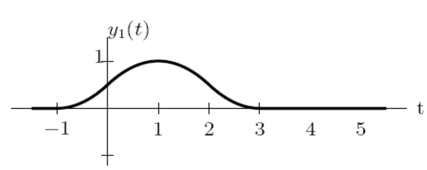
$$
\begin{aligned}
y_1(t) &= \frac{1}{2}(t+1)(t+1) \\
&= \frac{1}{2}(t^2 + 2t + 1)
\end{aligned}
$$

For the range $0 \le t < 1$, the area under the product is given by:

$$
\begin{aligned}
y_1(t) &= t(1-t) + \frac{1}{2}t(1-(1-t)) + \frac{1}{2} \\
&= t - t^2 + \frac{1}{2}t^2 + \frac{1}{2} \\
&= \frac{1}{2}(1 + 2t - t^2)
\end{aligned}
$$

Now both $x(t)$ and $h_1(t)$ are symmetric signals which are symmetric about $t = 0$ and $t = 1$ respectively. Therefore, the convolution of the two is symmetric about $t = 1$.

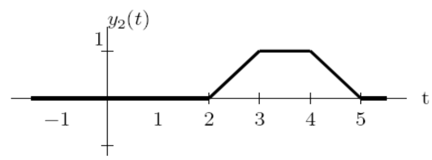
The plot for $y_1(t)$ looks like the following:



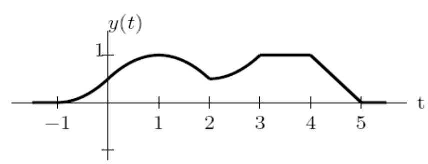
With the different regions of the curve as follows:

$$
\begin{aligned}
-1 < t < 0, \qquad & y_1(t) = \frac{1}{2}(t^2 + 2t + 1) \\
0 \le t < 1, \qquad & y_1(t) = \frac{1}{2}(1 + 2t - t^2) \\
1 \le t < 2, \qquad & y_1(t) = \frac{1}{2}(1 + 2t - t^2) \\
2 \le t < 3, \qquad & y_1(t) = \frac{1}{2}(t^2 - 6t + 9)
\end{aligned}
$$

The convolution with $h_2(t)$ is straightforward because it is a convolution with impulses. To do this, all we need is to center the triangle around both impulses and scale by the area under each impulse which in this case is 1. This gives the following plot for $y_2(t)$.



The final result is the sum of the two as follows:



The curved parts of the plot are given by the following expressions:

$$
\begin{aligned}
-1 < t < 0, \qquad & y(t) = \frac{1}{2}(t^2 + 2t + 1) \\
0 \le t < 1, \qquad & y(t) = \frac{1}{2}(1 + 2t - t^2) \\
1 \le t < 2, \qquad & y(t) = \frac{1}{2}(1 + 2t - t^2) \\
2 \le t < 3, \qquad & y(t) = \frac{1}{2}(t^2 - 4t + 5)
\end{aligned}
$$