BioE 2696/ECE 2695: Control Theory in Neuroscience
(3 Credits, Spring 2009)

Lecture 6: Control Theory III

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Zhi-Hong Mao
Assistant Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

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Outline

- Review of last lecture
- Stability
- Commonly used controllers

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Review of last lecture

- Steady-state gain (or dc gain) of a system
  - The system dc gain is the steady-state gain to a constant input for the case the output has a final value, and it is equal to the system transfer function evaluated at $s = 0$ (why?)

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Final value theorem

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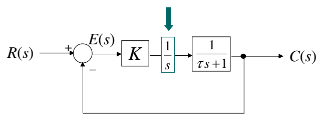
Review of last lecture

- Steady-state gain (or dc gain) of a system
- First-order systems
  - Leaky integrator and integrator
  - About system type



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Review of last lecture

- Steady-state gain (or dc gain) of a system
- First-order systems
- Second-order systems

$$G(s)=\frac{C(s)}{R(s)}=\frac{b_0}{s^2+a_1 s+a_0}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

? ?

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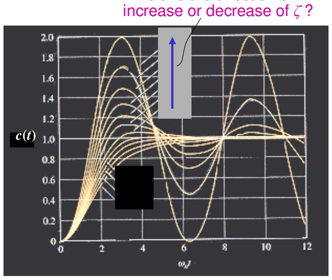
Review of last lecture

- Steady-state gain (or dc gain) of a system
- First-order systems
- Second-order systems

Is this the direction of increase or decrease of ζ ?



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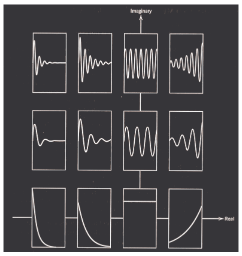
Review of last lecture

- Steady-state gain (or dc gain) of a system
- First-order systems
- Second-order systems
- Time response and pole locations



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Review of last lecture

- Steady-state gain (or dc gain) of a system
- First-order systems
- Second-order systems
- Time response and pole locations
- Frequency response: steady-state response of systems to sinusoidal inputs

$$G(j\omega) = |G(j\omega)|e^{j\phi(\omega)}$$

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Review of last lecture

- Steady-state gain (or dc gain) of a system
- First-order systems
- Second-order systems
- Time response and pole locations
- Frequency response: steady-state response of systems to sinusoidal inputs
  – Demo with a rubber band
  – Magnitude plot, phase plot, and Bode diagram

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Stability

- What do we usually mean by stability?
  - Examples of stable and unstable systems

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M2-F2 experiencing lateral oscillations in flight

[1] [2]

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Rest tremor in Parkinson’s disease

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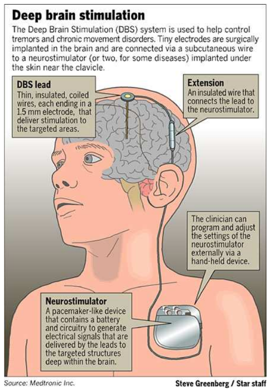

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Deep brain stimulation for the treatment of rest tremor in Parkinson's disease

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Stability

- What do we usually mean by stability?
- Bounded-input, bounded-output (BIBO) stability
  - A system is BIBO stable, if, for every bounded input, the output remains bounded for all time
  - The concept of BIBO stability is not appropriate to describe stability in all nonlinear systems

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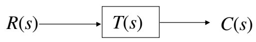
Stability

- What do we usually mean by stability?
- Bounded-input, bounded-output (BIBO) stability
- Criteria for the BIBO stability of linear time-invariant systems



$$T(s)=\frac{P(s)}{Q(s)},\quad Q(s)=a_n\prod_{i=1}^{n}(s-p_i)$$

$$C(s)=T(s)R(s)=\frac{P(s)}{a_n\prod_{i=1}^{n}(s-p_i)}R(s)=\frac{k_1}{s-p_1}+\cdots+\frac{k_n}{s-p_n}+C_r(s)$$

  - Some terms
    - Characteristic equation: $Q(s)=0$
    - System roots or system poles: $p_i,\ i=1,\ldots,n$
    - Forced response: $C_r(s)$ (the sum of terms, in the partial-fraction expansion, that originate in the poles of $R(s)$)

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Stability

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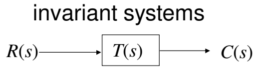

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  - Some terms
  - A linear time-invariant system is BIBO stable provided all roots of the system characteristic equation (poles of the closed-loop transfer function) lie in the left half of the s-plane (why?)

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PID control

- Proportional control
  - In proportional control, steady-state error tends to depend inversely upon proportional gain
  - Proportional control has a tendency to make a system faster
  - Proportional control does not change the order of the system

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PID control

- Proportional control
- Integral control
  - In integral control, steady-state error should be zero (prerequisite: the closed loop system has to be stable)
  - Integral control has a tendency to make a system slower and may even sacrifice stability
  - Integral control changes the order of the system

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PID control

- Proportional control
- Integral control
- Derivative control
  - Derivative control tends to increase the stability of the system
  - Derivative control tends to reduce the overshoot and improve the transient response
  - Derivative control changes the order of the system

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PID control

- Proportional control
- Integral control
- Derivative control

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
K_P	Decrease	Increase	Small change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small change	Decrease	Decrease	Small change

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PID control

- Proportional control
- Integral control
- Derivative control
- Another view on PID control
  - The proportional term gives the controller output a component that is a function of the present state of the system
  - The integrator output is determined by the past state of the system
  - The differentiator is a function of the slope of its input and thus can be considered to be a predictor of the future state of the system
  - The PID controller can viewed as giving control that is a function of the past, the present, and the predicted future

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An example of temperature control from hospital

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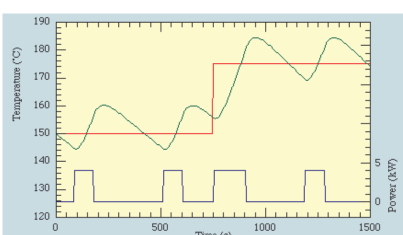
Temperature control using feedback



On-off control

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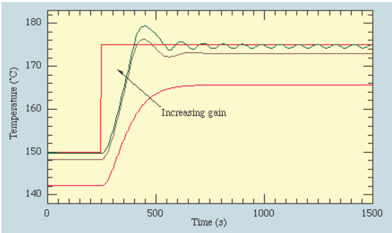
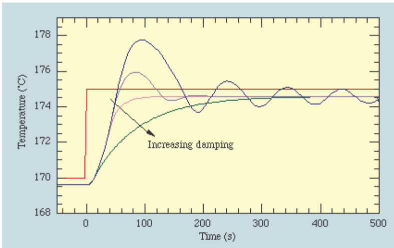
Temperature control using feedback



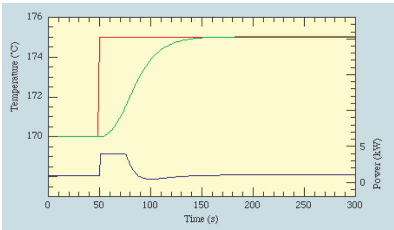
Proportional control

Temperature control using feedback



PD control

Temperature control using feedback



PID control

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