

Lecture 1: Signals and Systems I

January 5, 2009

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Outline

- Meaning of signals
- Common signals
- Size of a signal
- Representations of a signal

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Meaning of signals

- Definition
 - Any time-varying physical phenomenon that is intended to convey information is a **signal**
 - Examples: human voice, sign language, Morse code, traffic signals, voltages on telephone wires, etc.

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Meaning of signals

- Definition

- Any time-varying physical phenomenon that is intended to convey information is a signal
- Mathematically speaking, a **signal** is a function of time that is used to describe an observed physical variable of a physical process; it is an abstract mathematical description of the **observation**

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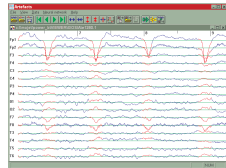
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- Examples of neural signals

EEG signals



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Meaning of signals

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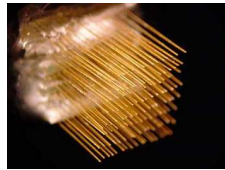
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- Examples of neural signals

ECoG recording



Implantable array of electrodes



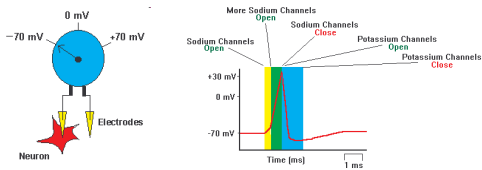
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- About noise

- Noise is like a signal in that it is a time-varying physical phenomenon, but it usually does not carry useful information and is considered undesirable
- Usefulness of noise in neural signal processing

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Meaning of signals

- Definition

- Domain of a signal: t 's for which it is defined

- Some common domains:

- All t : t can be any real number
- Nonnegative: $t \geq 0$
- t in some interval: $a \leq t \leq b$
- Discrete-time signals are defined for integer t , i.e., $t = 0, \pm 1, \pm 2, \dots$ (here t means sample time, not real time in seconds)

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Meaning of signals

- Definition
- Domain of a signal

- **Dimension of a signal**

- Real-valued or scalar signal (audio signals)
- Vector signal (images)
- Binary signal (trains of action potentials)

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Common signals

- A constant (or static or DC) signal: $x(t) = a$
- The **unit step** signal (sometimes denoted $1(t)$ or $u(t)$):

$$u(t) = 0 \text{ for } t < 0, u(t) = 1 \text{ for } t \geq 0$$

- The **unit ramp** signal:

$$x(t) = 0 \text{ for } t < 0, x(t) = t \text{ for } t \geq 0$$

- A **sinusoidal** signal:

$$x(t) = a \cos(\omega t + \phi)$$

↑ ↑ ↑
Signal parameters

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Common signals

- A constant (or static or DC) signal
- The unit step signal
- The unit ramp signal
- A sinusoidal signal

- **Pulse and impulse signals**

- The unit impulse signal, written $\delta(t)$, can be considered a limiting case of the pulse signal, $\delta_\Delta(t)$:

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & \text{if } 0 < t < \Delta \\ 0 & \text{otherwise} \end{cases}$$

$$\delta(t) = \lim_{\Delta \rightarrow 0} \delta_\Delta(t)$$

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Size of a signal

- Size of a signal x is measured in many ways, for example, if $x(t)$ is defined for $t \geq 0$:

- Integral square (or total energy): $\int_0^{\infty} x(t)^2 dt$

- Integral-absolute value: $\int_0^{\infty} |x(t)| dt$

- Peak (or maximum absolute value): $\max_{t \geq 0} |x(t)|$

- Root-mean-square (RMS) value: $\left(\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)^2 dt \right)^{1/2}$

- Average-absolute (AA) value:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |x(t)| dt$$

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Size of a signal

- Size of a signal x is measured in many ways

- Example: $x(t) = a \cos(\omega t + \phi)$

- Integral square (or total energy): $\int_0^{\infty} x(t)^2 dt = ?$

- Integral-absolute value: $\int_0^{\infty} |x(t)| dt = ?$

- Peak value: $\max_{t \geq 0} |x(t)| = ?$

- Root-mean-square (RMS) value: $\left(\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)^2 dt \right)^{1/2} = ?$

- Average-absolute (AA) value:

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |x(t)| dt = ?$$

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Representations of a signal

- In the time-domain, a signal is represented by a function of time
 - Data compression, basis functions, and transformations

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Representations of a signal

- In the time-domain, a signal is represented by a function of time
- In the frequency-domain: Fourier transform
 - A little history
 - Fourier and Lagrange
 - Any continuous periodic signal can be represented as the sum of properly chosen sinusoidal waves

Question: Why are sinusoids used instead of, for instance, square or triangular waves?

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Representations of a signal

- In the time-domain, a signal is represented by a function of time
- In the frequency-domain: Fourier transform
 - A little history
 - Fourier series for periodic-continuous signals



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Representations of a signal

- In the time-domain, a signal is represented by a function of time
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 - For a 2π -periodic function $x(t)$ integrable on $[-\pi, \pi]$, the numbers

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cos(nt) dt, \quad n \geq 0 \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \sin(nt) dt, \quad n \geq 1$$

are called Fourier coefficients of x . The infinite sum

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nt) + b_n \sin(nt)]$$

is called the Fourier series of x .

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Representations of a signal

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$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nt) + b_n \sin(nt)]$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \cos(nt) dt, n \geq 0 \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x(t) \sin(nt) dt, n \geq 1$$

- Introducing Euler's formula $e^{jnt} = \cos(nt) + j \sin(nt)$

Fourier series of f can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnt}$$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{-jnt} dt$$

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Representations of a signal

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$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jnt} \quad c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} x(t) e^{-jnt} dt$$

- Fourier transform for aperiodic-continuous signals

Forward transform $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

Inverse transform $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{+j\omega t} d\omega$

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 - Fourier series for periodic-continuous signals
 - Fourier transform for aperiodic-continuous signals
- Discrete Fourier transform (DFT)
 - The sequence of N complex numbers $x[0], \dots, x[N-1]$ is transformed into the sequence N complex numbers, $X[0], \dots, X[N-1]$ by the DFT according to the formula

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{2\pi j k n}{N}}, k = 0, \dots, N-1$$

- The inverse DFT (IDFT) is given by

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{\frac{2\pi j k n}{N}}, n = 0, \dots, N-1$$

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Representations of a signal

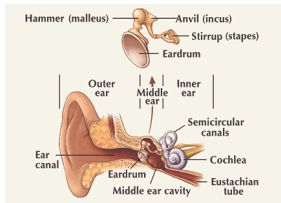
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- Discrete Fourier transform (DFT)
 - We can use the MATLAB function `fft` to calculate DFT
 - Example: Find DFT for $x[n] = 8\sin(\pi n/2)$ represented over the fundamental period $0 \leq n < N, N=4$

```
>> N = 4;
>> n = [0:N-1]';
>> x = 8*sin(pi*n/2);
>> X = fft(x);
>> X
```

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Representations of a signal

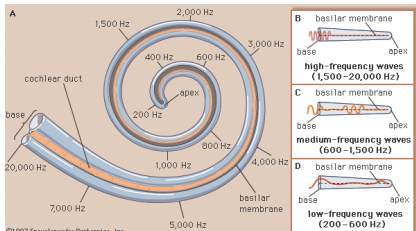
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References

- S. Boyd, Lecture Notes for EE102: Introduction to Signals and Systems, Stanford University.
- D. Heeger, Signals, Linear Systems, and Convolution, <http://white.stanford.edu/~heeger/ftp/handouts/convolution.pdf>
- D. K. Lindner, Introduction to Signals and Systems, McGraw-Hill, 1999.
- M. J. Roberts, Fundamentals of Signals and Systems, McGraw-Hill, 2008.
- S. W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing, 2nd Edition, California Technical Publishing, 1999.
- <http://faculty.washington.edu/chudler/ap.html>
- http://farm1.static.flickr.com/169/420008775_df516d18c4.jpg
- http://www.biosemi.com/pics/Praamstra_cap2_large.jpg
- <http://www.britannica.com/EBchecked/topic-art/123552/18100/Model-showing-the-distribution-of-frequencies-along-the-basilar-membrane>
- <http://www.coe.berkeley.edu/labnotes/1205/carmena.html>
- <http://www.cs.bris.ac.uk/~rafa/artifact/>
- <http://www.hearingcentral.com/howtheearworks.asp>

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