

Homework #5, BioE 2696/ECE2695 Control Theory in Neuroscience, Spring 2009

System Identification and Controller Design

In this assignment you will be asked to use the data provided to create a model of an unknown system. After modeling the system, you will then have to design a controller to make the system perform within a set of desired constraints.

System Identification

The data posted on the course website provides the step response of the unknown system to be modeled. More specifically, the data, $y(t)$ is the step response to a unit step input applied at $t = 0$, $u(t)$ or $1(t)$.

1. Plot the input/output relationship, i.e. plot $u(t)$ and $y(t)$

The unknown system will be referred to as the plant, or $Gp(s)$. We will assume that the plant can be modeled by a second order system in the following form:

$$Gp(s) = \frac{K_d \cdot \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where K_d = DC gain, ζ = damping coefficient, ω_n = natural frequency

2. Determine the unknown parameters of the system by analyzing the step response. Use the following equations and definitions:

$$\text{DC gain: } K_d = \frac{\text{steady state output}}{\text{step magnitude}}$$

$$\text{percent overshoot: } \%OS = \frac{100 \cdot (\text{max output} - \text{steady state output})}{\text{steady state output}}$$

$$\text{damping coefficient: } \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + (\ln(\%OS/100))^2}}$$

$$\text{damped natural frequency: } \omega_d = \frac{2\pi}{T_p}, \quad T_p = \text{period of sinusoidal component of output}$$

$$\text{natural frequency: } \omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}}$$

3. Plot the step response of your estimated model along with the data provided for comparison. How well does your second order model fit the data?

Controller Design

Implement a PID controller using the feedback control architecture shown in Figure 1.

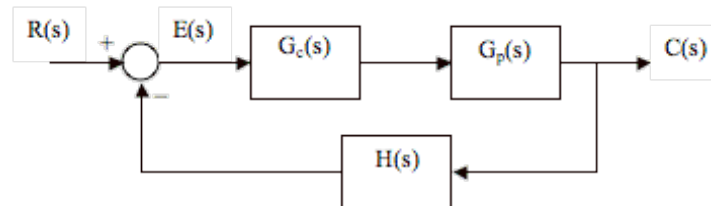


Figure 1: Negative feedback control system block diagram

where $G_c(s)$ is the PID controller, $G_p(s)$ is the plant, and $H(s)$ is the feedback signal. Let $H(s) = 1$, i.e. unity feedback.

PID controller transfer function: $G_c(s) = K_p + \frac{K_i}{s} + K_d s$

4. Design a PID controller to satisfy the following constraints:

- eliminate steady state error (steady state error < 1 %)
- eliminate overshoot (%OS < 1 %)
- settling time less than 6 seconds
(use the time it takes to reach 2% of steady state as the settling time)

Be sure to show the coefficients you have chosen to achieve the desired performance.

5. Plot the system step response using your PID feedback control design

Hint: Think about how each component of the PID controller affects the response of a second order system. There are many possible solutions so experiment with different values.