ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Course Review

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Outline

- Reminder from the first lecture
- Review of this course
  - Basic concepts
  - Basic techniques
- Exercises

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Questions for ourselves:

What have I learned from this course?
Which concepts, principles, and methods impressed me most in this course?
Which concepts, principles, and methods were most difficult to me?
What would I still remember about this course after ten years?

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Reminder from the first lecture

- This course covers:
  - System identification and real-time parameter estimation
  - Model-reference adaptive systems
  - Self-tuning regulators
  - Introduction to computational learning theory and learning in neural systems

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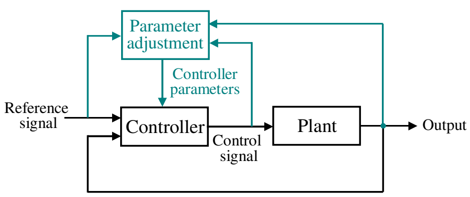
Basic concepts

- Adaptive control
  - Why adaptive control?
  - What is adaptive control?



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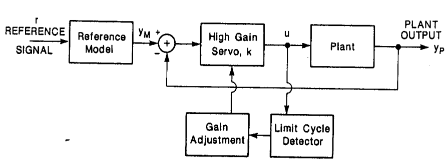
Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
  - Series high-gain scheme



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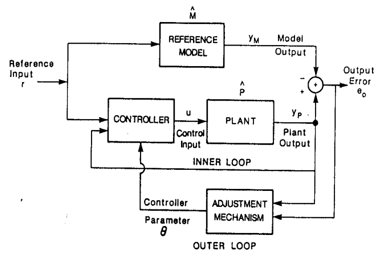
Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
  - Series high-gain scheme
  - Parallel scheme



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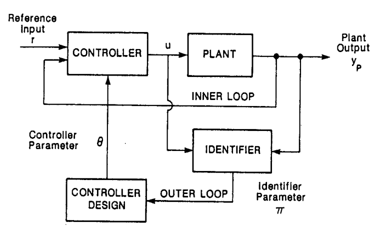
Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
  - Certainty equivalence principle



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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
  - Certainty equivalence principle
  - Direct and indirect adaptive control

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
    - Frequency response function
    - Monic, Hurwitz, stable, and minimum phase
    - Proper and strictly proper

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
    - Frequency response function
    - Monic, Hurwitz, stable, and minimum phase
    - Proper and strictly proper
    - Positive real and strictly positive real transfer functions

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
  - State-space description
  - Autonomous and non-autonomous (which one is time-invariant?)

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
  - State-space description
  - Autonomous and non-autonomous
  - Lipschitz condition (locally and globally)

$$\left| f(t, x_1) - f(t, x_2) \right| \le l \left| x_1 - x_2 \right|$$

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
  - State-space description
  - Autonomous and non-autonomous
  - Lipschitz condition (locally and globally)
  - Existence and uniqueness of solutions to a differential equation

$$\frac{dx}{dt} = f(t, x), \quad x(t_0) = x_0$$

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
  - State-space description
  - Autonomous and non-autonomous
  - Lipschitz condition (locally and globally)
  - Existence and uniqueness of solutions to a differential equation
  - Equilibrium point

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
  - Transfer function
  - State-space description
  - Autonomous and non-autonomous
  - Lipschitz condition (locally and globally)
  - Existence and uniqueness of solutions to a differential equation
  - Equilibrium point
  - Mixture of time-domain and frequency-domain notation

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
- Stability of dynamic systems
  - BIBO stability
  - Internal stability of an LTI system

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
- Stability of dynamic systems
  - BIBO stability
  - Internal stability of an LTI system
  - Stability in the sense of Lyapunov
    - Other stronger stability definitions

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
- Stability of dynamic systems
  - BIBO stability
  - Internal stability of an LTI system
  - Stability in the sense of Lyapunov
  - Lyapunov stability theory
    - Class K functions
    - l.p.d.f., p.d.f., and decrescent functions

Question (**Steiner's problem**): How to find a point inside a triangle that gives the shortest sum of distances to the vertices?

$$\frac{dv(t,x)}{dt} = \frac{\partial v(t,x)}{\partial t} + \frac{\partial v(t,x)}{\partial x} f(t,x)$$

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Basic concepts

- Adaptive control
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- Description of systems
- Stability of dynamic systems
  - BIBO stability
  - Internal stability of an LTI system
  - Stability in the sense of Lyapunov
  - Lyapunov stability theory
  - Control Lyapunov function

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
- Stability of dynamic systems
- Persistency of excitation (PE)

$$\alpha_2 I \ \geq \ \int_{t_0}^{t_0+\delta} w(\tau)\, w^T(\tau)\, d\tau \ \geq \ \alpha_1 I \qquad \text{for all } t_0 \geq 0$$

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Basic concepts

- Adaptive control
- Model reference adaptive control (MRAC)
- Self-tuning control (STC)
- Description of systems
- Stability of dynamic systems
- Persistency of excitation (PE)
  - PE and exponential stability
    - Let w: $\mathbf{R}_+ \rightarrow \mathbf{R}^{2n}$ be piecewise continuous and PE, then the differential equation

$$\dot{\phi}(t) = -g\, w(t)\, w^T(t)\, \phi(t) \qquad g > 0$$

is globally exponential stable

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Basic techniques

- Basic identification methods
  - Frequency domain approach

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Basic techniques

- Basic identification methods
  - Frequency domain approach
  - Time domain approach
    - SPM and DPM

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Basic techniques

- Basic identification methods
  - Frequency domain approach
  - Time domain approach
    - SPM and DPM
    - Gradient algorithm

$$e_1(t) = \theta^T w(t) - y_p(t) = \left[\theta^T - \theta^{*T}\right] w(t)$$

Objective: minimize $e_1^2(t)$

Gradient: $\dfrac{\partial}{\partial\theta}\left(e_1^2\right) = 2e_1 \dfrac{\partial}{\partial\theta}\left(e_1\right) = 2e_1 w$

Parameter update law: $\dfrac{d\theta}{dt} = -g e_1 w$

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Basic techniques

- Basic identification methods
  - Frequency domain approach
  - Time domain approach
    - SPM and DPM
    - Gradient algorithm
    - Least-squares algorithm

Objective: minimize the integral-squared-error (ISE)

$$\text{ISE} = \int_0^t \left[\theta^T(\tau) w(\tau) - y_p(\tau)\right]^2 d\tau$$

$$\theta_{LS}(t) = \left[\int_0^t w(\tau)\, w^T(\tau)\, d\tau\right]^{-1} \left[\int_0^t w(\tau)\, y_p(\tau)\, d\tau\right]$$

Least-squares estimate

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Basic techniques

- Basic identification methods
  - Frequency domain approach
  - Time domain approach
    - SPM and DPM
    - Gradient algorithm
    - Least-squares algorithm

Recursive formulation

$$\dot\theta(t) = -P(t) w(t) \left[\theta^T(t) w(t) - y_p(t)\right] \quad \theta(0) = \theta_0$$

$$\dot P(t) = -P(t) w(t) w^T(t) P(t) \quad P(0) = P^T(0) = P_0$$

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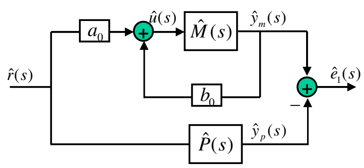
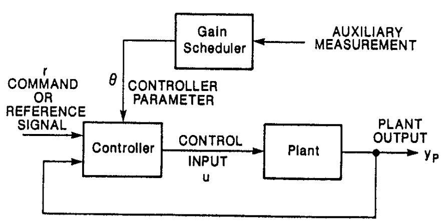
Basic techniques

- Basic identification methods
  - Frequency domain approach
  - Time domain approach
    - SPM and DPM
    - Gradient algorithm
    - Least-squares algorithm
    - Model reference identification



Basic techniques

- Basic identification methods
- Gain scheduling



Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
  - MIT rule

Objective: Adjust the parameter θ to minimize

$$J(\theta) = \frac{1}{2} e_0^2(\theta)$$

Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
  - MIT rule

$$\frac{d\theta}{dt} = -\gamma e_0(\theta)\frac{\partial}{\partial\theta}\left(y_p(\theta)\right)$$

MIT rule: an implementation of the gradient update by replacing the unknown parameters in $\partial y_p(\theta)/\partial\theta$ by their estimates at time t

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Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
  - MIT rule
  - Lyapunov design
    - Basic steps:

    (1) Find a controller structure

    (2) Derive the error equation

    (3) Find a Lyapunov function and use it to derive a parameter updating law such that the error will go to zero

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Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
  - MIT rule
  - Lyapunov design
    - Basic steps
    - Lyapunov design for state-space systems

If all eigenvalues of A_m lie in the open left-half plane, then for each symmetric positive definite matrix Q there exists a unique symmetric positive definite matrix P such that

$$A_m^T P + PA_m = -Q.$$

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Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
  - MIT rule
  - Lyapunov design
    - Basic steps
    - Lyapunov design for state-space systems

$$\frac{de}{dt} = A_m e + \psi(\theta - \theta^*)$$

$$v(e,\theta) = \frac{g}{2} e^T P e + \frac{1}{2}(\theta - \theta^*)^T(\theta - \theta^*)$$

$$\frac{d\theta}{dt} = -g\psi^T P e$$

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Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
  - MIT rule
  - Lyapunov design
    - Basic steps
    - Lyapunov design for state-space systems
    - Using output feedback

(1) Find a controller structure that admits perfect output tracking

(2) Derive an error model of the form

$$\varepsilon = \hat{G}_1(s)\{w^T(t)(\theta - \theta^*)\}$$

where $\hat{G}_1$ is an SPR transfer function

(3) Use the parameter adjustment law $\frac{d\theta}{dt} = -g\, w\, \varepsilon$

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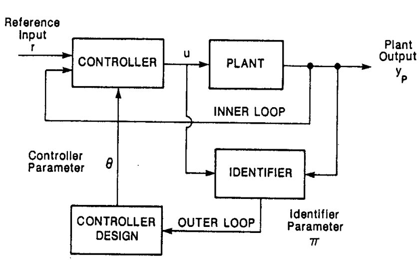
Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
- STC



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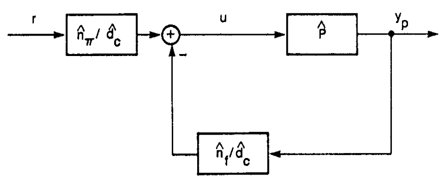
Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
- STC
  - System identification
  - Controller design
    - Output feedback



Basic techniques

- Basic identification methods
- Gain scheduling
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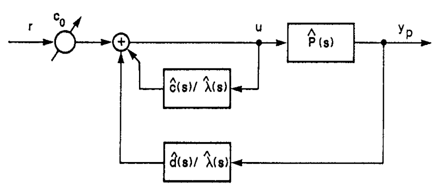
  - System identification
  - Controller design
    - Output feedback



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Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
- STC
  - System identification
  - Controller design
    - Output feedback
    - State feedback

Theorem (multivariable case)**:** All eigenvalues of $A - BK$ can be assigned arbitrarily (provided complex conjugate eigenvalues are assigned in pairs) by selecting a real constant matrix K if and only if $(A,\ B)$ is controllable.

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Basic techniques

- Basic identification methods
- Gain scheduling
- MRAC
- STC
- Practical aspects
  - Sampling and pre/post-filtering
  - Estimator implementation
    - Simple model structure
    - Data filters and excitation
    - Parameter tracking
  - Interaction of estimation and control
    - Computational delay
    - Integral action
    - Compatibility for identification and control

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