ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Lecture 9: Self-Tuning Control (I)

November 3, 2008

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Outline

- Homework problems
- Comparison between self-tuning control (STC) and MRAC
- Parametric models
- Parameter identification (revisited)

Homework problems

- Homework 7.1: Consider the third-order plant

$$y = \hat{G}(s)u$$

  where

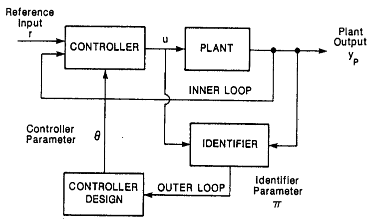
$$\hat{G}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}.$$

  Obtain parametric models for the plant in the form of SPM and DPM where $\theta^* = [b_2, b_1, b_0, a_2, a_1, a_0]^T$.

- Homework 7.2: In the above problem, if a_0, a_1, a_2 are known, obtain a parametric model for the plant in terms of $\theta^* = [b_2, b_1, b_0]^T$. If b_0, b_1, b_2 are known, obtain a parametric model for the plant in terms of $\theta^* = [a_2, a_1, a_0]^T$.

Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
  - A self-tuning controller is a controller obtained by coupling a controller with an online (recursive) parameter estimator



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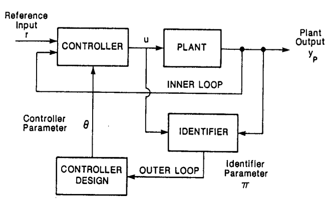

Question: What is the “certainty equivalence” principle?

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Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
  - A self-tuning controller is a controller obtained by coupling a controller with an online (recursive) parameter estimator
  - Parameter estimator
    - An parameter estimator serves a role of finding a set of parameters that fits the available input-output data from a plant

Question: What is the difference between the parameter estimation in STC and parameter adaptation in MRAC?

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Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
  - A self-tuning controller is a controller obtained by coupling a controller with an online (recursive) parameter estimator
  - Parameter estimator
    - An parameter estimator serves finding a set of parameters that fits the available input-output data from a plant
    - Many techniques are available to estimate the unknown parameters of the plant

Question: Could you name a few of these parameter estimation techniques?

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Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
  - A self-tuning controller is a controller obtained by coupling a controller with an online (recursive) parameter estimator
  - Parameter estimator
  - Controller
    - Many control techniques can be used in the design of a controller: pole-placement, PID, and LQR (linear quadratic control)

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Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
  - A self-tuning controller is a controller obtained by coupling a controller with an online (recursive) parameter estimator
  - Parameter estimator
  - Controller
  - A variety of self-tuning controllers can be obtained by coupling different control and estimation schemes

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Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
  - A self-tuning controller is a controller obtained by coupling a controller with an online (recursive) parameter estimator
  - Parameter estimator
  - Controller
  - A variety of self-tuning controllers can be obtained by coupling different control and estimation schemes
  - A basic scheme in STC is to estimate the plant parameters first and then compute the controller parameters; such a scheme is often called indirect adaptive control

Question: What is direct adaptive control?

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Comparison between STC and MRAC

- Definition of self-tuning controllers (STC)
- Relations between MRAC and STC methods
  - Both MRAC and STC systems have an inner loop for control and an outer loop for parameter estimation
  - Compared with MRAC, STC are more flexible because of the possibility of coupling various controllers with various estimators (i.e., the separation of control and estimation)

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  - The stability and convergence of STC are generally more difficult to guarantee, often requiring the signals in the system to be sufficiently rich so that the estimated parameters converge to the true parameters

Question: What can we do if the input signals are not rich enough?

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Comparison between STC and MRAC

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  - The stability and convergence of STC are generally more difficult to guarantee, often requiring the signals in the system to be sufficiently rich so that the estimated parameters converge to the true parameters
  - In MRAC, the stability and tracking error convergence are usually guaranteed regardless of the richness of the signals (why?)

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Parametric models

- Static parametric model (SPM)

A reminder:

$$y_p(t) = \theta^{*T} w(t) = w^T(t)\theta^*$$

$$e_1(t) = \theta^T w(t) - y_p(t) = \left(\theta^T - \theta^{*T}\right) w(t)$$

Nominal identifier parameter

Identification error

Adaptive identifier parameter

Linear error equation

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Parametric models

- Static parametric model (SPM)
  - An example

$$\dot{x} = -x + ax + bu$$

$$x = \frac{1}{s+1}(ax+bu) = a\frac{1}{s+1}x + b\frac{1}{s+1}u$$

$$\theta^* = [a,b]^T, w = \left[\frac{1}{s+1}x, \frac{1}{s+1}u\right]^T$$

$$x = \theta^{*T} w = w^T \theta^*$$

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Parametric models

- Static parametric model (SPM)
  - An example
  - Definition

$$z = \theta^{*T} w = w^T \theta^*$$

This is called an SPM

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Parametric models

- Static parametric model (SPM)
  - An example
  - Definition
  - An example from my research (with Prof. Ruiping Xia, Sushant Tare, and Chris Sprague)

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Parametric models

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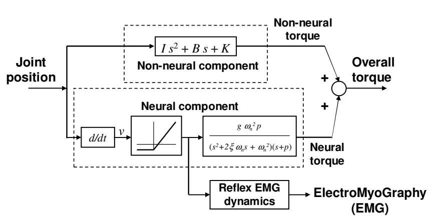

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Parametric models

- Static parametric model (SPM)
  - An example
  - Definition
  - An example from my research (with Prof. Ruiping Xia, Sushant Tare, and Chris Sprague)

$$(Is^2 + Bs + K)x = u$$

$$u = I(s^2 x) + B(sx) + K(x)$$

$$\theta^* = [I, B, K]^T, w = \left[s^2 x, sx, x \right]^T$$

$$u = \theta^{*T} w$$

Question: What is the problem with this formulation?

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Parametric models

- Static parametric model (SPM)
  - An example
  - Definition
  - An example from my research (with Prof. Ruiping Xia, Sushant Tare, and Chris Sprague)

$$u = I(s^2 x) + B(sx) + K(x)$$

$$\Lambda(s) = (s+\lambda)^2, \ \lambda > 0$$

$$\frac{1}{\Lambda(s)} u = I\left(\frac{s^2}{\Lambda(s)} x\right) + B\left(\frac{s}{\Lambda(s)} x\right) + K\left(\frac{1}{\Lambda(s)} x\right)$$

$$z = \frac{1}{\Lambda(s)} u, \theta^* = [I, B, K]^T, w = \left[\frac{s^2}{\Lambda(s)} x, \frac{s}{\Lambda(s)} x, \frac{1}{\Lambda(s)} x \right]^T$$

$$z = \theta^{*T} w$$

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Parametric models

- Static parametric model (SPM)
  - An example
  - Definition
  - An example from my research (with Prof. Ruiping Xia, Sushant Tare, and Chris Sprague)

An alternative SPM for this problem

$$\frac{1}{\Lambda(s)} u = I\left(\frac{s^2}{\Lambda(s)} x\right) + B\left(\frac{s}{\Lambda(s)} x\right) + K\left(\frac{1}{\Lambda(s)} x\right)$$

Question: What are the values of z, θ^*, w ?

$$\left(\frac{s^2}{\Lambda(s)} x\right) = \frac{1}{I}\left(\frac{1}{\Lambda(s)} u\right) - \frac{B}{I}\left(\frac{s}{\Lambda(s)} x\right) - \frac{K}{I}\left(\frac{1}{\Lambda(s)} x\right)$$

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Parametric models

- Static parametric model (SPM)
- Dynamic parametric model (DPM)
  - An example

$$\dot{x} = -x + ax + bu$$

$$\downarrow$$

$$x = \frac{1}{s+1}(ax+bu) = \frac{1}{s+1}\left([a \;\; b] \begin{bmatrix} x \\ u \end{bmatrix} \right)$$

$$\theta^* = [a,b]^T, w = [x,u]^T$$

$$x = \hat{M}(s)\left(\theta^{*T} w\right)$$

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Parametric models

- Static parametric model (SPM)
- Dynamic parametric model (DPM)
  - An example
  - Definition

$$z = \hat{M}(s)\left(\theta^{*T} w\right)$$

This is called a DPM

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Parametric models

- Static parametric model (SPM)
- Dynamic parametric model (DPM)
  - An example
  - Definition
  - Another example

$$\dot{x} = -2x + af(x) + bg(x)u$$

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Parametric models

- Static parametric model (SPM)
- Dynamic parametric model (DPM)
  - An example
  - Definition
  - Another example

$$\dot{x} = -2x + af(x) + bg(x)u \;\rightarrow\; x = \frac{1}{s+2}[af(x) + bg(x)u]$$

$$z = x, \hat{M}(s) = \frac{1}{s+2},$$
$$\theta^* = [a, b]^T, w = [f(x), g(x)u]^T$$

$$z = \hat{M}(s)\left(\theta^{*T} w\right)$$

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Parametric models

- Static parametric model (SPM)
- Dynamic parametric model (DPM)
  - An example
  - Definition
  - Another example

$$\dot{x} = -2x + af(x) + bg(x)u$$

Question: What if we want $\hat{M}(s)$ to be a design transfer function with a pole, say at $\lambda > 0$?

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Parameter identification (revisited)

- Gradient algorithm
  - For SPM:

$$e_1(t) = \theta^T w(t) - \theta^{*T} w(t) = \left(\theta^T - \theta^{*T}\right) w(t)$$

Objective: minimize $e_1^2(t)$

Gradient: $\frac{\partial}{\partial\theta}\left(e_1^2\right) = 2e_1 \frac{\partial}{\partial\theta}\left(e_1\right) = 2e_1 w$

Parameter update law: $\frac{d\theta}{dt} = -g e_1 w$

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Parameter identification (revisited)

- Gradient algorithm
  - For SPM
  - For DPM:

$$e_1(t) = \hat{M}(s)\left[\theta^T w(t) - \theta^{*T} w(t)\right] = \hat{M}(s)\left[\left(\theta^T - \theta^{*T}\right) w(t)\right]$$

$$\text{Parameter update law: } \frac{d\theta}{dt} = -g e_1 w$$

Remark: If $\hat{M}(s)$ is SPR, the above gradient update law has similar properties when e_1 is defined by the DPM and SPM?

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Parameter identification (revisited)

- Gradient algorithm
- Least-square algorithms

Objective: minimize the integral-squared-error (ISE)

$$\text{ISE} = \int_0^t \left[\theta^T(\tau) w(\tau) - z(\tau)\right]^2 d\tau$$

$$\theta_{LS}(t) = \left[\int_0^t w(\tau)\, w^T(\tau)\, d\tau\right]^{-1} \left[\int_0^t w(\tau)\, z(\tau)\, d\tau\right]$$

Least-squares estimate

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Parameter identification (revisited)

- Gradient algorithm
- Least-square algorithms

Recursive formulation

$$\dot{\theta}(t) = -P(t)w(t)\left[\theta^T(t)w(t) - z(t)\right], \quad \theta(0) = \theta_0$$

$$\dot{P}(t) = -P(t)w(t)w^T(t)P(t), \quad P(0) = P^T(0) = P_0$$

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References

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- P. Ioannou and B. Fidan, Adaptive Control Tutorial, SIAM, 2006.
- S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence, and Robustness, Prentice-Hall, 1989.
- J.-J. E. Slotine and W. Li, Applied Nonlinear Control, Prentice Hall, 1991.

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