ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Lecture 8: Model Reference Adaptive Control (IV)

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Outline

- Homework problems
- Review of last lecture
- More about PR and SPR transfer functions
- Adaptation of a feedforward gain
- Output feedback

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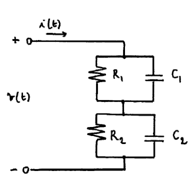
Homework problems

- Problem 6.1: For the circuit shown on the right, find conditions on R_1, R_2, C_1, C_2 (all greater than 0) such that the impedance $\hat{Z}(s) = \hat{V}(s)/\hat{I}(s)$ is SPR.



- **Extra credit problem:** Let $\hat{G}(s)$ be a rational, SPR transfer function. Let $y(t)$ be the step response associated with $\hat{G}(s)$. Show that

$$\left(\frac{dy}{dt}\right)_{t=0} > 0 \quad \text{and} \quad \left(\frac{d^2 y}{dt^2}\right)_{t=0} < 0.$$

(Hint: Use the Kalman-Yakubovich lemma.)

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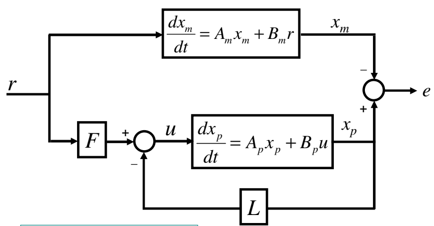
Review of last lecture



$$\frac{de}{dt} = A_m e + \psi(\theta - \theta^*)$$

$$\frac{d\theta}{dt} = -g\psi^T P e$$

$$v(e,\theta) = \frac{g}{2} e^T P e + \frac{1}{2}(\theta - \theta^*)^T(\theta - \theta^*)$$

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More about PR and SPR transfer functions

- Definition
  - A rational transfer function $\hat{G}$ with real coefficients is positive real (PR) if

$$\mathrm{Re}\,\hat{G}(s) \geq 0 \quad \text{for} \quad \mathrm{Re}\, s > 0$$

  - A transfer function $\hat{G}$ is strictly positive real (SPR) if $\hat{G}(s-\varepsilon)$ is positive real for some real $\varepsilon > 0$

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More about PR and SPR transfer functions

- Definition
- Origin of these concepts
  - The definition of PR and SPR transfer functions is derived from network theory
  - A PR (SPR) rational transfer function can be realized as the driving point impedance of a passive (dissipative) network
  - A passive (dissipative) network has a driving point impedance that is rational and PR (SPR)
  - A passive network is one that does not generate energy (e.g., a circuit consisting only of resistors, capacitors, and inductors)

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More about PR and SPR transfer functions

- Definition
- Origin of these concepts
- Properties of PR and SPR transfer functions
  - A rational proper transfer function $\hat{G}(s)$ is PR if and only if

    (i) $\hat{G}(s)$ is real for real s;

    (ii) $\hat{G}(s)$ is analytic in $\operatorname{Re} s > 0$, and the poles on the $j\omega$-axis are simple and such that the associated residues are real and positive;

    (iii) for all ω for which $s = j\omega$ is not a pole of $\hat{G}(s)$, one has $\operatorname{Re}\hat{G}(j\omega) \geq 0$.

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More about PR and SPR transfer functions

- Definition
- Origin of these concepts
- Properties of PR and SPR transfer functions
  - A rational proper transfer function $\hat{G}(s)$ is SPR iff

    (i) $\hat{G}(s)$ is real for real s;

    (ii) $\hat{G}(s)$ is analytic in $\operatorname{Re} s > 0$, and has no poles or zeros on the imaginary axis;

    (iii) for all ω, $\operatorname{Re}\hat{G}(j\omega) \geq 0$.

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More about PR and SPR transfer functions

- Definition
- Origin of these concepts
- Properties of PR and SPR transfer functions
  - $\hat{G}(s)$ is PR (SPR) iff $1/\hat{G}(s)$ is PR (SPR)
  - If $\hat{G}(s)$ is SPR, then its relative degree is no greater than 1, and its zeros and poles lie in $\operatorname{Re} s < 0$
  - If the relative degree of $\hat{G}(s)$ is greater than 1, then $\hat{G}(s)$ is not PR

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More about PR and SPR transfer functions

- Definition
- Origin of these concepts
- Properties of PR and SPR transfer functions
  - A necessary condition for $\hat{G}(s)$ to be PR is that the Nyquist plot of $\hat{G}(j\omega)$ lie in the right half complex plane, which implies that the phase shift in the output of a system with transfer function $\hat{G}(s)$ in response to a sinusoidal input is less than 90^o

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Adaptation of a feedforward gain

- The system
  - Plant: $k\hat{G}(s)$, where $\hat{G}(s)$ is known but k is unknown
  - Model: $k_0\hat{G}(s)$
  - Controller: $u = \theta r$

Review question: What is the parameter adjustment law derived using the MIT rule?

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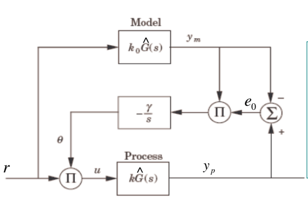
Adaptation of a feedforward gain

- The system
  - Plant: $k\hat{G}(s)$, where $\hat{G}(s)$ is known but k is unknown
  - Model: $k_0\hat{G}(s)$
  - Controller: $u = \theta r$



$$e_0 = y_p - y_m = k\hat{G}(s)\theta r - k_0\hat{G}(s)r$$

$$\frac{\partial e_0}{\partial \theta} = k\hat{G}(s)r = \frac{k}{k_0} y_m$$

$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_0} y_m e_0 = -\gamma y_m e_0$$

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Adaptation of a feedforward gain

- The system
- Error equation

$$e_0 = [k\hat{G}(s)\theta - k_0\hat{G}(s)]r = k\hat{G}(s)(\theta - \theta^*)r$$

↓ State-space equation

$$\frac{dx}{dt} = Ax + B(\theta - \theta^*)r$$
$$e_0 = Cx$$

Adaptation of a feedforward gain

- The system
- Error equation
- Lyapunov function

$\dot{x} = Ax$ asymptotically stable ⟷ **?** There exist positive definite matrices P and Q such that

$$A^T P + PA = -Q$$

Adaptation of a feedforward gain

- The system
- Error equation
- Lyapunov function

$$\frac{dx}{dt} = Ax + B(\theta - \theta^*)r, \quad e_0 = Cx$$

$$A^T P + PA = -Q$$

$$v(x,\theta) = \frac{1}{2} g x^T P x + \frac{1}{2}(\theta - \theta^*)^2$$

Exercise: Derive the parameter adjustment law.

Adaptation of a feedforward gain

- The system
- Error equation
- Lyapunov function

$$\frac{dx}{dt} = Ax + B(\theta - \theta^*)r$$
$$e_0 = Cx$$

$$A^T P + PA = -Q$$

$$v(x,\theta) = \frac{1}{2} g x^T P x + \frac{1}{2}(\theta - \theta^*)^2$$

$$\frac{dv}{dt} = \frac{g}{2}\left((Ax + Br(\theta - \theta^*))^T Px + x^T P(Ax + Br(\theta - \theta^*))\right) + (\theta - \theta^*)\frac{d\theta}{dt}$$
$$= -\frac{g}{2} x^T Q x + (\theta - \theta^*)\left(\frac{d\theta}{dt} + grB^T Px\right)$$

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Adaptation of a feedforward gain

- The system
- Error equation
- Lyapunov function

$$\frac{dv}{dt} = -\frac{g}{2} x^T Q x + (\theta - \theta^*)\left(\frac{d\theta}{dt} + grB^T Px\right)$$

- Parameter adjustment law

$$\frac{d\theta}{dt} = -g\ r\ B^T P x$$

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Adaptation of a feedforward gain

- The system
- Error equation
- Lyapunov function
- Parameter adjustment law
  - Output feedback

$$\frac{d\theta}{dt} = -g\ r\ B^T P x$$

If P can be chosen such that

$$B^T P = C$$

$$\frac{d\theta}{dt} = -g\ r\ B^T P x = -g\ r\ Cx = -g\ r\ e_0$$

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Adaptation of a feedforward gain

- The system
- Error equation
- Lyapunov function
- Parameter adjustment law
- Kalman-Yakubovich lemma
  - Let the LTI system $\dot{x} = Ax + Bu, \quad y = Cx$

    be completely controllable and observable. The transfer function $\hat{G}(s) = C(sI - A)^{-1}B$

    is strictly positive real if and only if there exist positive definite matrices P and Q such that

    $A^T P + PA = -Q$ and $B^T P = C.$

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Output feedback

- Basic steps in the Lyapunov design using output feedback
  - Find a controller structure that admits perfect output tracking
  - Derive an error model of the form

    $$\varepsilon = \hat{G}_1(s)\{w^T(t)(\theta - \theta^*)\}$$

    where $\hat{G}_1$ is an SPR transfer function
  - Use the parameter adjustment law

    $$\frac{d\theta}{dt} = -g\, w\, \varepsilon$$

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References

- K. J. Astrom and B. Wittenmark, Adaptive Control, 2nd Edition, Addison-Wesley, 1995.
- P. Ioannou and B. Fidan, Adaptive Control Tutorial, SIAM, 2006.
- S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence, and Robustness, Prentice-Hall, 1989.

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