ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Lecture 7: Model Reference Adaptive Control (III)

October 20, 2008

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Outline

- Homework problems
- Review of the midterm exam
- Lyapunov design for general linear systems
- Positive real (PR) and strictly positive real (SPR) transfer functions

Homework problems

- Problem 5.1: An integrator b/s is to be controlled by a controller $u = \theta_1 r - \theta_2 y_p$. The desired response model is given by $b_m/(s + a_m)$. Derive, using the Lyapunov theory, a parameter update law guaranteeing that the error $e = y_p - y_m$ goes to zero.
- Problem 5.2: Determine whether the following transfer functions are positively real and whether they are strictly positively real:

$$\frac{1}{s},\ \frac{1}{s+1},\ \left(\frac{1}{s+1}\right)^2,\ \frac{s+c}{(s+a)(s+b)}$$

where $a > 0, b > 0, a + b > c > 0.$

Review of the midterm exam

- Stability of LTI systems
- Lyapunov stability
- Persistently exciting
- The MIT rule
- Lyapunov design for MRAC

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Lyapunov design for general linear systems

- Basic steps
  - Find a controller structure
  - Derive the error equation
  - Find a Lyapunov function and use it to derive a parameter updating law such that the error will go to zero

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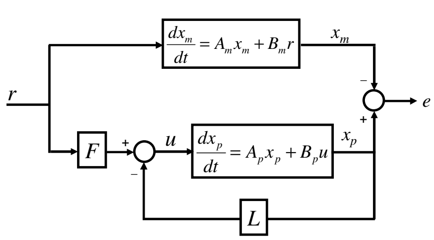
Lyapunov design for general linear systems

- Basic steps
- State-space systems



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Lyapunov design for general linear systems

- Basic steps
- State-space systems
  - Plant $$\frac{dx_p}{dt} = A_p x_p + B_p u$$
  - Reference model $$\frac{dx_m}{dt} = A_m x_m + B_m r$$
  - Controller $$u = Fr - Lx_p$$
  - Model match $$\frac{dx_p}{dt} = (A_p - B_p L)x_p + B_p Fr = A_c(\theta)x_p + B_c(\theta)r$$ $$\text{with } A_c(\theta^*) = A_m \text{ and } B_c(\theta^*) = B_m$$

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Lyapunov design for general linear systems

- Basic steps
- State-space systems

$$\frac{dx_p}{dt} = A_p x_p + B_p u \quad \frac{dx_m}{dt} = A_m x_m + B_m r \quad u = Fr - Lx_p$$

$$\frac{dx_p}{dt} = (A_p - B_p L)x_p + B_p Fr = A_c(\theta)x_p + B_c(\theta)r$$
$$\text{with } A_c(\theta^*) = A_m \text{ and } B_c(\theta^*) = B_m$$

- Error equation

$$e = x_p - x_m$$

$$\frac{de}{dt} = A_m e + \psi(\theta - \theta^*)$$ ← How to derive this?

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Lyapunov design for general linear systems

- Basic steps
- State-space systems
- Error equation

$$\frac{de}{dt} = A_m e + \psi(\theta - \theta^*)$$

- Lyapunov design
  - Lyapunov function

$$v(e,\theta) = \frac{g}{2} e^T Pe + \frac{1}{2}(\theta - \theta^*)^T (\theta - \theta^*)$$

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Lyapunov design for general linear systems

- Basic steps
- State-space systems
- Error equation
- Lyapunov design
  - Lyapunov function
  - Lyapunov lemma

$$\frac{de}{dt} = A_m e + \psi(\theta - \theta^*)$$

$$v(e,\theta) = \frac{g}{2} e^T P e + \frac{1}{2}(\theta - \theta^*)^T(\theta - \theta^*)$$

If all eigenvalues of A_m lie in the open left-half plane, then for each symmetric positive definite matrix Q there exists a unique symmetric positive definite matrix P such that

$$A_m^T P + P A_m = -Q.$$

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Lyapunov design for general linear systems

- Basic steps
- State-space systems
- Error equation
- Lyapunov design
  - Lyapunov function
  - Lyapunov lemma
  - Parameter adjustment law

$$\frac{de}{dt} = A_m e + \psi(\theta - \theta^*)$$

$$v(e,\theta) = \frac{g}{2} e^T P e + \frac{1}{2}(\theta - \theta^*)^T(\theta - \theta^*)$$

$$\frac{d\theta}{dt} = -g\psi^T P e$$

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Lyapunov design for general linear systems

- Basic steps
- State-space systems
- Error equation
- Lyapunov design
- An example

From last lecture:

$$\frac{dy_p}{dt} = -a_p y_p + k_p u$$
$$\frac{dy_m}{dt} = -a_m y_m + k_m u$$
$$u = c_0 r + d_0 y_p$$

$$e_0 = y_p - y_m$$
$$\phi = \begin{bmatrix} \phi_r \\ \phi_y \end{bmatrix} = \begin{bmatrix} c_0 - c_0^* \\ d_0 - d_0^* \end{bmatrix}$$
$$c_0^* = \frac{k_m}{k_p}, \quad d_0^* = \frac{a_p - a_m}{k_p}$$

$$\dot{\phi} = \begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_y \end{bmatrix} = \begin{bmatrix} -g e_0 r \\ -g e_0 y_p \end{bmatrix}$$

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Positive real and strictly positive real transfer functions

- Definition
  - A rational transfer function $\hat{G}$ with real coefficients is positive real (PR) if

$$\operatorname{Re}\hat{G}(s) \geq 0 \quad \text{for} \quad \operatorname{Re} s \geq 0$$

  - A transfer function $\hat{G}$ is strictly positive real (SPR) if $\hat{G}(s-\varepsilon)$ is positive real for some real $\varepsilon > 0$

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References

- K. J. Astrom and B. Wittenmark, Adaptive Control, 2nd Edition, Addison-Wesley, 1995.
- S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence, and Robustness, Prentice-Hall, 1989.
- J.-J. E. Slotine and W. Li, Applied Nonlinear Control, Prentice Hall, 1991.

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