ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Lecture 6: Model Reference Adaptive Control (II)

October 6, 2008

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Outline

- Announcements
- Homework problems
- Review of last lecture
- Examples of the MIT rule (cont'd)
- Stability of the MIT rule
- Lyapunov design (design based on Lyapunov stability theory)

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Announcements

- Midterm exam will be given on October 14 (Tuesday); October 13 (Monday) is Fall Break
  - Please pay attention to my lecture notes
  - Please pay attention to the homework problems
- Extra office hour session for this week
  - Friday 4:00 pm—6:00 pm
- Possible personal emergency

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Homework problems

- No homework problems for this week
- Past homework problems
  - Problem 1.10(b)
  - Problem 0.3

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Homework problems

- No homework problems for this week
- Past homework problems
  - Problem 4.1: Consider the plant $\hat{P}(s) = \frac{1}{s(s+a)}$ where a is an unknown parameter. Determine a controller that can give the closed-loop system
  $$\hat{M}(s) = \frac{\omega^2}{s^2 + 2\varsigma\omega s + \omega^2}.$$
  - Problem 4.2: Control an integrator $\hat{P}(s) = b/s$ with a controller $u = \theta_1 r - \theta_2 y_p$. The desired response model is given by
  $$\hat{M}(s) = b_m/(s + a_m).$$
  Derive the MIT rule for parameter update.

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Review of last lecture

- What is the MIT rule?
- Mixture of time-domain and frequency-domain notations

Exercise: Rewrite the following equation using time-domain notation only

$$\frac{\partial e_0}{\partial \theta} = k \frac{b_m}{s + a_m} r.$$

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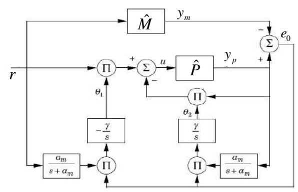
Review of last lecture



Question: Is this a linear system?

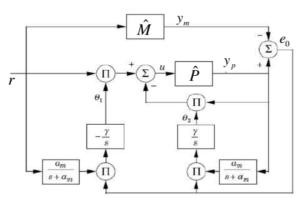
Exercise: Write down the parameter-update equations for the above system.

Review of last lecture

Plant: $\frac{dy_p}{dt} = -ay_p + bu$

Model: $\frac{dy_m}{dt} = -a_m y_m + b_m r$

Controller: $u = \theta_1 r - \theta_2 y_p$



$e_0 = y_p - y_m$

$\frac{\partial e_0}{\partial \theta_1} = \frac{b}{s+a+b\theta_2} r$

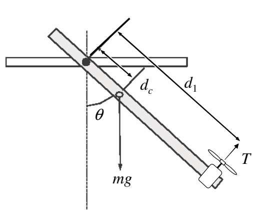
$\frac{\partial e_0}{\partial \theta_1} = -\frac{b^2\theta_1}{(s+a+b\theta_2)^2} r = -\frac{b}{s+a+b\theta_2} y_p$

$s+a+b\theta_2 \approx s+a+b\theta_2^* = s+a_m$

MIT rule:
$$\frac{d\theta_1}{dt} = -\gamma\left(\frac{a_m}{s+a_m} r\right) e_0$$
$$\frac{d\theta_2}{dt} = \gamma\left(\frac{a_m}{s+a_m} y_p\right) e_0$$

Question: Why do we use a_m here?

Example 3



System dynamics

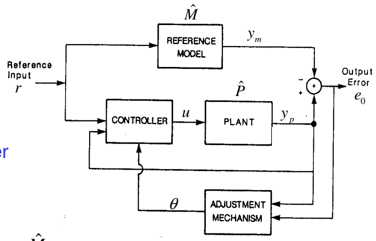
$$J\ddot{\theta} + c\dot{\theta} + mgd_c \sin\theta = d_1 T$$

$$\frac{\theta(s)}{T(s)} = \frac{d_1}{Js^2 + cs + mgd_c}$$

$$\frac{\theta(s)}{T(s)} = \frac{1.89}{s^2 + 0.0389s + 10.77}$$

Example 3

Form of controller



$$u = \theta_1 r - \theta_2 y_p$$

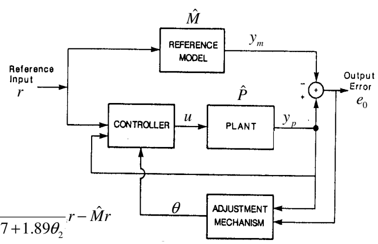
$$e_0 = y_p - y_m = \hat{P}u - \hat{M}r$$

$$y_p = \hat{P}u = \left(\frac{1.89}{s^2 + 0.0389s + 10.77}\right)\left(\theta_1 r - \theta_2 y_p\right)$$

$$y_p = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

Example 3

Gradients



$$e_0 = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r - \hat{M}r$$

$$\frac{\partial e_0}{\partial \theta_1} = \frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

$$\frac{\partial e_0}{\partial \theta_2} = -\frac{1.89^2\theta_1}{\left(s^2 + 0.0389s + 10.77 + 1.89\theta_2\right)^2} r = -\frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} y_p$$

$$y_p = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

Example 3

$$\frac{\partial e_0}{\partial \theta_1} = \frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

$$\frac{\partial e_0}{\partial \theta_2} = -\frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} y_p$$

Approximation of the gradients

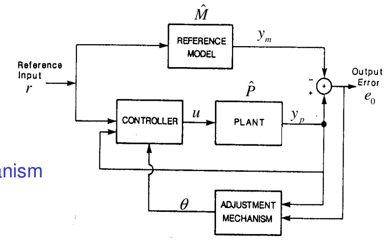
$$y_p = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

$$s^2 + 0.0389s + 10.77 + 1.89\theta_2 \approx s^2 + a_{1m}s + a_{0m}$$

$$\frac{\partial e_0}{\partial \theta_1} = \frac{1.89}{s^2 + a_{1m}s + a_{0m}} r$$

$$\frac{\partial e_0}{\partial \theta_2} = -\frac{1.89}{s^2 + a_{1m}s + a_{0m}} y_p$$

Example 3



Adjustment mechanism

$$\frac{d\theta_1}{dt} = -\gamma' \frac{\partial e_0}{\partial \theta_1} e_0 = -\gamma \left(\frac{a_{0m}}{s^2 + a_{1m}s + a_{0m}} r \right) e_0$$

$$\frac{d\theta_2}{dt} = -\gamma' \frac{\partial e_0}{\partial \theta_2} e_0 = \gamma \left(\frac{a_{0m}}{s^2 + a_{1m}s + a_{0m}} y_p \right) e_0$$

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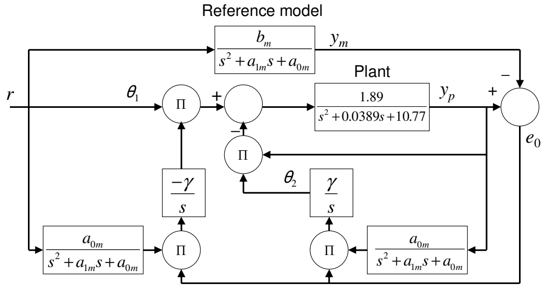
Example 3



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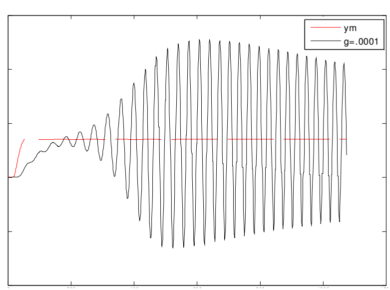
Example 3



Remark: The plant almost goes unstable. This response is largely due to the near instability of the open loop system. Tuning of gamma and changing the reference model did not alleviate this problem.

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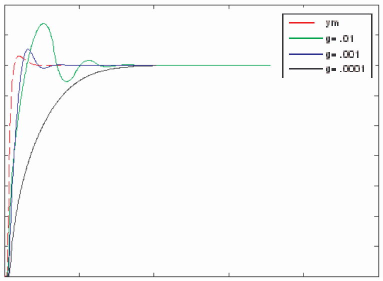
Example 3



Remark: For this specific example, introducing PD control may stabilize the system dynamics.

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Example 3

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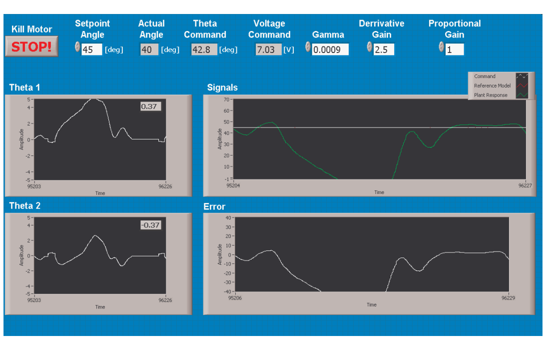
Example 3



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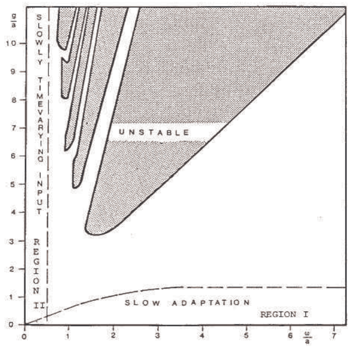
Stability of the MIT rule



$$\hat{P}(s) = \frac{k}{s+1}, \hat{M}(s) = \frac{k_0}{s+1}$$
$$r = \sin(\omega t)$$

Remark: In this case, for small enough ω, the MIT rule is stable for all g, while for small enough g, it is stable for all ω. Otherwise, the dynamic behavior of the algorithm is quite complex, with interleaving of the stable and unstable regions.

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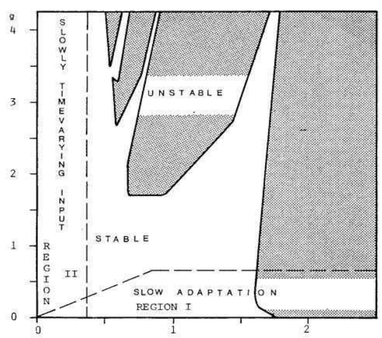
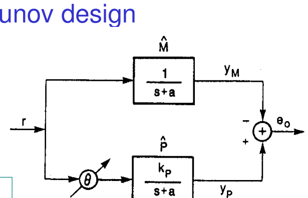
Stability of the MIT rule

The case where $\hat{P}(s)$ and $\hat{M}(s)$ do not share the same set of poles and zeros



Lynapunov design

- Example 4



$$v(e_0, \phi) = e_0^2 + k_p \phi^2$$

Question: How to determine the update law for ϕ?

$$\dot{y}_p = -a\, y_p + k_p \theta r$$
$$\dot{y}_m = -a\, y_m + r = -a\, y_m + k_p \theta^* r$$
$$\dot{e}_0 = -a\, e_0 + k_p (\theta - \theta^*) r$$
$$\phi = \theta - \theta^*$$

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Lynapunov design

- Example 4

$$v(e_0,\phi) = e_0^2 + k_p\phi^2$$
$$\dot{v}(e_0,\phi) = -2a e_0^2 + 2k_p e_0 \phi r + 2k_p \phi \dot{\phi}$$
$$\dot{\phi} = -e_0 r$$

Question: Can we guarantee the convergence of e_0? Can we guarantee the convergence of ϕ?

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Lynapunov design

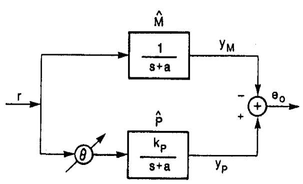
- Example 4

The MIT rule

$$\dot{\theta} = -g e_0 y_m$$

Rule derived from Lyapunov stability theory

$$\dot{\theta} = \dot{\phi} = -e_0 r$$



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Lynapunov design

- Example 5
  - Plant:

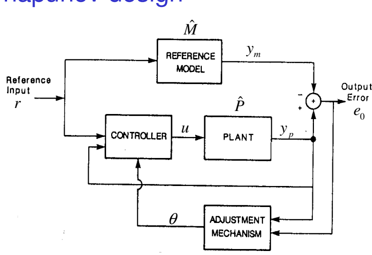
$$\frac{dy_p}{dt} = -a_p y_p + k_p u$$

  - Model:

$$\frac{dy_m}{dt} = -a_m y_m + k_m r$$

  - Controller:

$$u = c_0 r + d_0 y_p$$



Question: What are the nominal values of c_0 and d_0?

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Lynapunov design

- Example 5

The system

$$\frac{dy_p}{dt} = -a_p y_p + k_p u$$
$$\frac{dy_m}{dt} = -a_m y_m + k_m u$$
$$u = c_0 r + d_0 y_p$$

Parameters

$$e_0 = y_p - y_m$$
$$\phi = \begin{bmatrix} \phi_r \\ \phi_y \end{bmatrix} = \begin{bmatrix} c_0 - c_0^* \\ d_0 - d_0^* \end{bmatrix}$$
$$c_0^* = \frac{k_m}{k_p}, \quad d_0^* = \frac{a_p - a_m}{k_p}$$

Exercise: Derive equation for e_0.

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Lynapunov design

- Example 5

Lyapunov function

$$v(e_0, \phi_r, \phi_y) = \frac{e_0^2}{2} + \frac{k_p}{2g}(\phi_r^2 + \phi_y^2)$$

Exercise: Design parameter update laws to ensure convergence of the Lyapunov function

$$\dot{e}_0 = -a_m e_0 + k_p(\phi_r r + \phi_y y_p)$$
$$\dot{\phi} = \begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_y \end{bmatrix} = ?$$

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Lynapunov design

- Example 5

$$v(e_0, \phi_r, \phi_y) = \frac{e_0^2}{2} + \frac{k_p}{2g}(\phi_r^2 + \phi_y^2)$$
$$\dot{v} = -a_m e_0^2 + k_p \phi_r e_0 r + k_p \phi_y e_0 y_p + \frac{k_p}{g}\phi_r \dot{\phi}_r + \frac{k_p}{g}\phi_y \dot{\phi}_y$$
$$\dot{\phi} = \begin{bmatrix} \dot{\phi}_r \\ \dot{\phi}_y \end{bmatrix} = \begin{bmatrix} -g e_0 r \\ -g e_0 y_p \end{bmatrix}$$

Question: Can we guarantee the convergence of e_0?
Can we guarantee the convergence of ϕ?

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Lynapunov design

- Example 5

Lyapunov function

$$v(e_0,\phi_r,\phi_y)=\frac{e_0{}^2}{2}+\frac{k_p}{2g}(\phi_r{}^2+\phi_y{}^2)$$

Rule derived from Lyapunov stability theory

$$\dot{\phi}=\begin{bmatrix}\dot{\phi}_r\\ \dot{\phi}_y\end{bmatrix}=\begin{bmatrix}-ge_0 r\\ -ge_0 y_p\end{bmatrix}$$

The MIT Rule

$$\dot{\theta}_1=-\gamma\left(\frac{a_m}{s+a_m}r\right)e_0$$

$$\dot{\theta}_2=\gamma\left(\frac{a_m}{s+a_m}y_p\right)e_0$$

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References

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