ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Lecture 5: Model Reference Adaptive Control (I)

September 29, 2008

Instructor: Zhi-Hong Mao
Assistant Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

1

Outline

- Homework problems
- Review of last lecture
- Concept of MRAC
- The MIT rule
- Examples
- Stability of the MIT rule

2

Homework problems

- Homework 4
  - Problem 1: Consider the plant $\hat{P}(s) = \dfrac{1}{s(s+a)}$ where a is an unknown parameter. Determine a controller that can give the closed-loop system
  $$\hat{M}(s) = \frac{\omega^2}{s^2 + 2\varsigma\omega s + \omega^2}.$$
  - Problem 2: Control an integrator $\hat{P}(s) = b/s$ with a controller $u = \theta_1 r - \theta_2 y_p$. The desired response model is given by
  $$\hat{M}(s) = b_m/(s + a_m).$$
  Derive the MIT rule for parameter update.

3

Homework problems

- About last homework

If A has distinct eigenvalues, then the equation $\dot{x} = Ax$ is marginally stable if and only if all eigenvalues of A have zero or negative real parts.

The equation $\dot{x} = Ax$ is marginally stable if and only if all eigenvalues of A have zero or negative real parts and those with zero real parts are simple roots of the minimal polynomial of A.

The equation $\dot{x} = Ax$ is asymptotically stable if and only all eigenvalues of A have negative real parts.

4

Review of last lecture

- Persistency of excitation (PE)
  - Definition
    - A vector w: $\mathbf{R}_+ \to \mathbf{R}^{2n}$ is persistently exciting (PE) if there exists $\alpha_1, \alpha_2, \delta > 0$ such that

$$\alpha_2 I \;\geq\; \int_{t_0}^{t_0+\delta} w(\tau)\, w^T(\tau)\, d\tau \;\geq\; \alpha_1 I \qquad \text{for all } t_0 \geq 0$$

5

Review of last lecture

- Persistency of excitation (PE)
  - Definition
    - A vector w: $\mathbf{R}_+ \to \mathbf{R}^{2n}$ is persistently exciting (PE) if there exists $\alpha_1, \alpha_2, \delta > 0$ such that

$$\alpha_2 \;\geq\; \int_{t_0}^{t_0+\delta} (w^T(\tau)\, x)^2\, d\tau \;\geq\; \alpha_1 \qquad \text{for all } t_0 \geq 0,\ |x| = 1$$

6

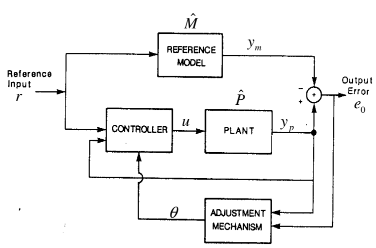
Review of last lecture

- Persistency of excitation (PE)
  - Definition
  - PE and exponential stability
    - Let $w: \mathbf{R}_+ \rightarrow \mathbf{R}^{2n}$ be piecewise continuous and PE, then the differential equation

$$\dot{\phi}(t) = -g\, w(t)\, w^T(t)\, \phi(t) \qquad g > 0$$

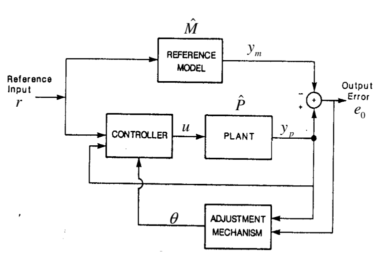
is globally exponential stable

Concept of MRAC



- Design controller to drive plant response to mimic ideal response (error $\rightarrow 0$)
- Designer chooses the reference model, controller structure, and tuning gains for adjustment mechanism

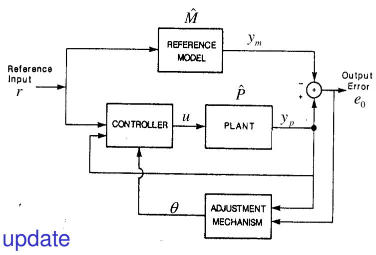
MIT rule



Objective: Adjust the parameter θ to minimize

$$J(\theta) = \frac{1}{2} e_0^2(\theta)$$

MIT rule



Gradient update

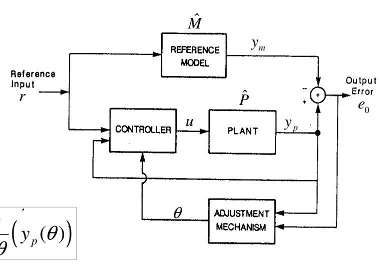
$$\frac{d\theta}{dt} = -\gamma \frac{\partial}{\partial\theta}\left(\frac{1}{2}e_0^2(\theta)\right) = -\gamma e_0(\theta)\frac{\partial}{\partial\theta}\big(e_0(\theta)\big) = -\gamma e_0(\theta)\frac{\partial}{\partial\theta}\big(y_p(\theta)\big)$$

Adaptation gain (sometimes we use symbol g)

Sensitivity function

10

MIT rule

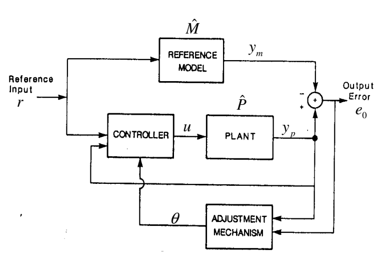


$$\frac{d\theta}{dt} = -\gamma e_0(\theta)\frac{\partial}{\partial\theta}\big(y_p(\theta)\big)$$

MIT rule: an implementation of the gradient update by replacing the unknown parameters in $\partial y_p(\theta)/\partial\theta$ by their estimates at time t

11

MIT rule



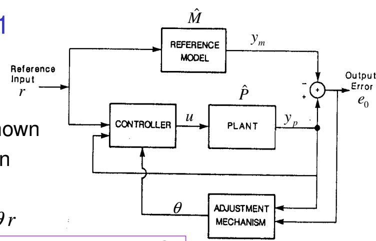
Question: What is the MIT rule if we choose to minimize

$$J(\theta) = |e_0|?$$

12

Example 1

- Plant: $k\hat{G}(s)$ where $\hat{G}(s)$ is known but k is unknown
- Model: $k_0\hat{G}(s)$
- Controller: $u = \theta r$

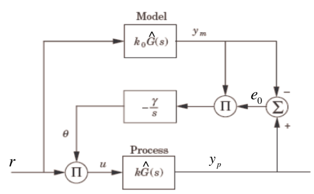


Question:
$$e_0(\theta) = ?$$
$$\frac{\partial}{\partial \theta}\left(e_0(\theta)\right) = ?$$
$$\frac{d\theta}{dt} = ?$$

13

Example 1

- Plant: $k\hat{G}(s)$ where $\hat{G}(s)$ is known but k is unknown
- Model: $k_0\hat{G}(s)$
- Controller: $u = \theta r$



$$e_0 = y_p - y_m = k\hat{G}(s)\theta r - k_0\hat{G}(s)r$$
$$\frac{\partial e_0}{\partial \theta} = k\hat{G}(s)r = \frac{k}{k_0} y_m$$
$$\frac{d\theta}{dt} = -\gamma' \frac{k}{k_0} y_m e_0 = -\gamma y_m e_0$$

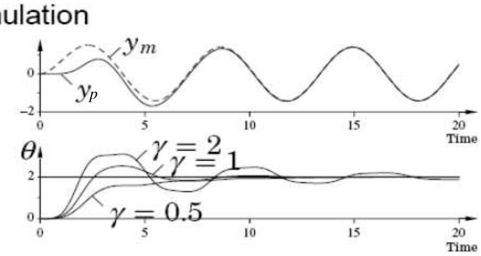
A mixture of time-domain and frequency-domain notations (such hybrid notation is common in the adaptive control literature—it will save us the definition of intermediate variables)

14

Example 1

$$\hat{G}(s) = \frac{1}{s+1}$$
$k = 1$ and $k_0 = 2$
$r = \sin(t)$

Simulation



Example 2

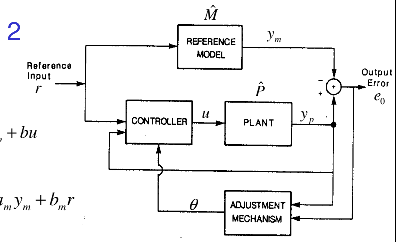
- Plant:

$$\frac{dy_p}{dt} = -ay_p + bu$$

- Model:

$$\frac{dy_m}{dt} = -a_m y_m + b_m r$$

- Controller: $u = \theta_1 r - \theta_2 y_p$



Question:

$$\frac{\partial}{\partial\theta}\left(e_0(\theta)\right) = ? \qquad \frac{d\theta}{dt} = ?$$

Example 2

- Plant: $\frac{dy_p}{dt} = -ay_p + bu$
- Model: $\frac{dy_m}{dt} = -a_m y_m + b_m r$
- Controller: $u = \theta_1 r - \theta_2 y_p$

Question: Can these formulas be used directly in the MIT rule?

$$e_0 = y_p - y_m$$

$$\frac{\partial e_0}{\partial\theta_1} = \frac{b}{s + a + b\theta_2} r$$

$$\frac{\partial e_0}{\partial\theta_2} = -\frac{b^2\theta_1}{(s + a + b\theta_2)^2} r = -\frac{b}{s + a + b\theta_2} y_p$$

Example 2

- Plant: $\frac{dy_p}{dt} = -ay_p + bu$
- Model: $\frac{dy_m}{dt} = -a_m y_m + b_m r$
- Controller: $u = \theta_1 r - \theta_2 y_p$

$$e_0 = y_p - y_m$$

$$\frac{\partial e_0}{\partial\theta_1} = \frac{b}{s + a + b\theta_2} r$$

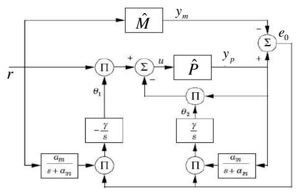
$$\frac{\partial e_0}{\partial\theta_1} = -\frac{b^2\theta_1}{(s + a + b\theta_2)^2} r = -\frac{b}{s + a + b\theta_2} y_p$$

Approximation:

$$s + a + b\theta_2 \approx s + a + b\theta_2^* = s + a_m$$

Example 2

- Plant: $\frac{dy_p}{dt} = -ay_p + bu$
- Model: $\frac{dy_m}{dt} = -a_m y_m + b_m r$
- Controller: $u = \theta_1 r - \theta_2 y_p$



$$e_0 = y_p - y_m$$
$$\frac{\partial e_0}{\partial \theta_1} = \frac{b}{s+a+b\theta_2} r$$
$$\frac{\partial e_0}{\partial \theta_1} = -\frac{b^2\theta_1}{(s+a+b\theta_2)^2} r = -\frac{b}{s+a+b\theta_2} y_p$$

$$s+a+b\theta_2 \approx s+a+b\theta_2^* = s+a_m$$

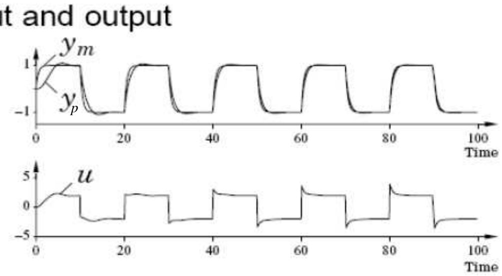
MIT rule:
$$\frac{d\theta_1}{dt} = -\gamma\left(\frac{a_m}{s+a_m} r\right)e_0$$
$$\frac{d\theta_2}{dt} = \gamma\left(\frac{a_m}{s+a_m} y_p\right)e_0$$

19

Example 2

$a = 1$ and $b = 0.5$
$a_m = b_m = 2$
$\gamma = 1$

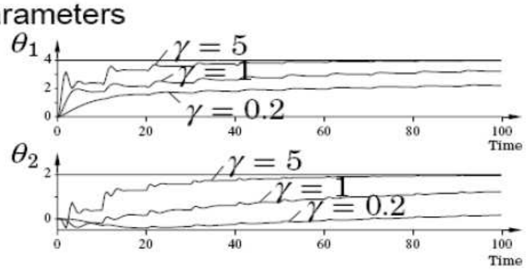
Input and output



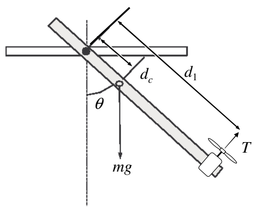
Example 2

$a = 1$ and $b = 0.5$
$a_m = b_m = 2$
$\theta_1^* = 4$ and $\theta_2^* = 2$

Parameters



Example 3



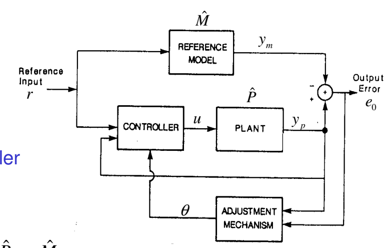
System dynamics

$$J\ddot{\theta} + c\dot{\theta} + mgd_c \sin\theta = d_1 T$$

$$\frac{\theta(s)}{T(s)} = \frac{d_1}{Js^2 + cs + mgd_c}$$

$$\frac{\theta(s)}{T(s)} = \frac{1.89}{s^2 + 0.0389s + 10.77}$$

Example 3



Form of controller

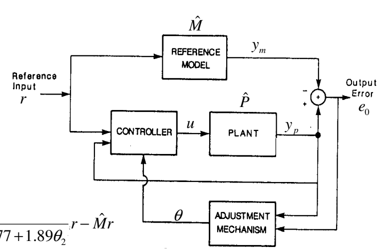
$$u = \theta_1 r - \theta_2 y_p$$

$$e_0 = y_p - y_m = \hat{P}u - \hat{M}r$$

$$y_p = \hat{P}u = \left(\frac{1.89}{s^2 + 0.0389s + 10.77}\right)(\theta_1 r - \theta_2 y_p)$$

$$y_p = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

Example 3



Gradients

$$e_0 = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r - \hat{M}r$$

$$\frac{\partial e_0}{\partial \theta_1} = \frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

$$\frac{\partial e_0}{\partial \theta_2} = -\frac{1.89^2\theta_1}{(s^2 + 0.0389s + 10.77 + 1.89\theta_2)^2} r = -\frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} y_p$$

$$y_p = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

Example 3

$$\frac{\partial e_0}{\partial \theta_1} = \frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

$$\frac{\partial e_0}{\partial \theta_2} = -\frac{1.89}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} y_p$$

Approximation of the gradients

$$y_p = \frac{1.89\theta_1}{s^2 + 0.0389s + 10.77 + 1.89\theta_2} r$$

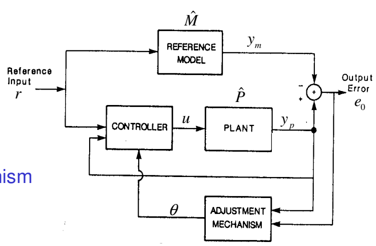
$$s^2 + 0.0389s + 10.77 + 1.89\theta_2 \approx s^2 + a_{1m}s + a_{0m}$$

$$\frac{\partial e_0}{\partial \theta_1} = \frac{1.89}{s^2 + a_{1m}s + a_{0m}} r$$

$$\frac{\partial e_0}{\partial \theta_2} = -\frac{1.89}{s^2 + a_{1m}s + a_{0m}} y_p$$

Example 3

Adjustment mechanism

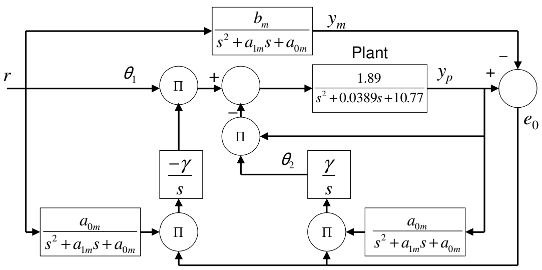


$$\frac{d\theta_1}{dt} = -\gamma' \frac{\partial e_0}{\partial \theta_1} e_0 = -\gamma \left(\frac{a_{0m}}{s^2 + a_{1m}s + a_{0m}} r \right) e_0$$

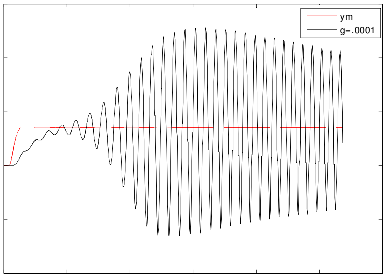
$$\frac{d\theta_2}{dt} = -\gamma' \frac{\partial e_0}{\partial \theta_2} e_0 = \gamma \left(\frac{a_{0m}}{s^2 + a_{1m}s + a_{0m}} y_p \right) e_0$$

Example 3

Reference model



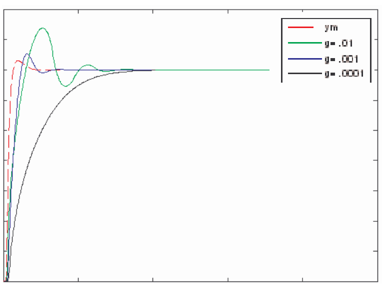
Example 3



Remark: The plant almost goes unstable. This response is largely due to the near instability of the open loop system. Tuning of gamma and changing the reference model did not alleviate this problem.

28

Example 3



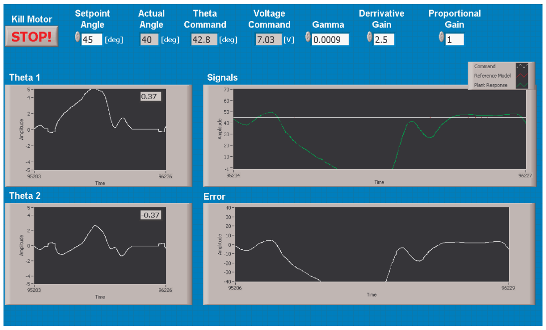
Remark: For this specific example, introducing PD control may stabilize the system dynamics.

29

Example 3

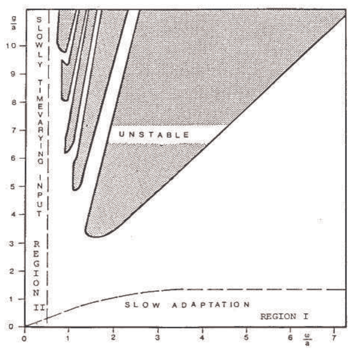
30

Example 3



31

Stability of the MIT rule



$\hat{P}(s) = \frac{k}{s+1}, \hat{M}(s) = \frac{k_0}{s+1}$

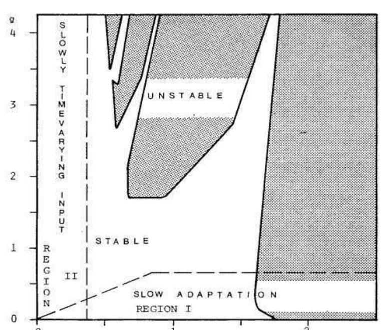
$r = \sin(\omega t)$

Remark: In this case, for small enough ω, the MIT rule is stable for all g, while for small enough g, it is stable for all ω. Otherwise, the dynamic behavior of the algorithm is quite complex, with interleaving of the stable and unstable regions.

32

Stability of the MIT rule

The case where $\hat{P}(s)$ and $\hat{M}(s)$ do not share the same set of poles and zeros



References

- B. D. O. Anderson, Failures of adaptive control theory and their resolution, Communications in Information and Systems, vol. 5, no. 1, pp. 1-20, 2005.
- K. J. Astrom and B. Wittenmark, Adaptive Control, 2nd Edition, Addison-Wesley, 1995.
- S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence, and Robustness, Prentice-Hall, 1989.
- K. Sevcik, Model reference adaptive control—survey of control systems [presentation]. Available online: http://www.pages.drexel.edu/~kws23/tutorials/MRAC/MRAC.html
- J.-J. E. Slotine and W. Li, Applied Nonlinear Control, Prentice Hall, 1991.

34