ECE 2695: Adaptive Control (3 Credits, Fall 2008)

Lecture 10: Self-Tuning Control (II)

November 10, 2008

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1

Outline

- Homework problems
- Course survey next week
- Review of last lecture
- Controller design: output feedback
- Controller design: state feedback
- An example

2

Homework problems

- Problem 8.1: Suppose you have a pendulum with frequency ω_0 and a transfer function given by $1/(s^2+\omega_0^2)$. Find the control law that the closed-loop transfer function is $1/(s+3\omega_0)^2$.

- Problem 8.2: A state-space description for the above pendulum system is given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \quad y = [1 \quad 0]x.$$

Find the control law $u = r - [k_1 \quad k_2]x$ (i.e., determine the values of k_1 and k_2) that places both of the closed-loop poles of the system to be $-3\omega_0$.

3

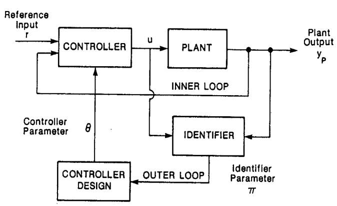
Course survey next week

- I have scheduled a survey for you to complete during class next week. Please give your serious consideration to this process, which will provide me with important information for planning this course in the future. I would like feedback from *all* members of the class, so please give me your independent opinions and suggestions.
- Thank you very much!

4

Review of last lecture

- Comparison between STC and MRAC



Question: What is the "certainty equivalence" principle?

Question: What is the difference between the parameter estimation in STC and parameter adaptation in MRAC?

5

Review of last lecture

- Comparison between STC and MRAC

Question: What are direct adaptive control and indirect adaptive control?

Question: Comparing STC and MRAC, which one has more flexible structure (and why)? Which one is easier to guarantee stability and convergence (and why)?

6

Review of last lecture

- Comparison between STC and MRAC
- Parametric models: SPM and DPM

$$z = \theta^{*T} w = w^T \theta^*$$

$$z = \hat{M}(s)\left(\theta^{*T} w\right)$$

7

Review of last lecture

- Comparison between STC and MRAC
- Parametric models: SPM and DPM

Exercise: Consider the nonlinear system

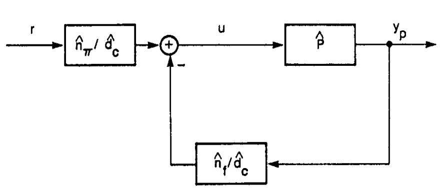
$$\ddot{x} + 2\dot{x} + x = a_1 f_1(x) + a_2 f_2(x) + b_1 g_1(x)u + b_2 g_2(x)u$$

where a_1, a_2, b_1, b_2 are unknown constants and $x, f_1(x), f_2(x)$, $g_1(x)$, $g_2(x)$, u are available for measurement. Express the unknown parameters in the form of SPM and DPM.

8

Controller design: output feedback

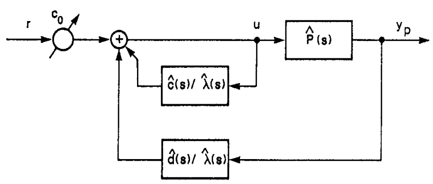
- Controller structures



9

Controller design: output feedback

- Controller structures



10

Controller design: output feedback

- Controller structures
- Plant assumptions
  - The plant is a single-input, single-output (SISO), linear time invariant (LTI) system, described by a transfer function

$$\frac{\hat{y}_p(s)}{\hat{u}(s)} = \hat{P}(s) = k_p \frac{\hat{n}_p(s)}{\hat{d}_p(s)}$$

where $\hat{n}_p(s), \hat{d}_p(s)$ are monic, coprime polynomials of degree m and n, respectively

11

Controller design: output feedback

- Controller structures
- Plant assumptions

$$\frac{\hat{y}_p(s)}{\hat{u}(s)} = \hat{P}(s) = k_p \frac{\hat{n}_p(s)}{\hat{d}_p(s)}$$

  - The plant is strictly proper and minimum phase (not necessarily stable)
  - The sign of the so-called high-frequency gain k_p is known and, without loss of generality, we will assume $k_p > 0$

12

Controller design: output feedback

- Controller structures
- Plant assumptions
- Controller design objective
  - To design a controller such that the closed-loop system has a desired transfer function

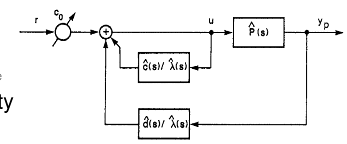
$$\hat{M}(s) = k_m \frac{\hat{n}_m(s)}{\hat{d}_m(s)}$$

where $\hat{n}_m(s), \hat{d}_m(s)$ are monic, coprime polynomials of degree m and n, respectively (that is, the same degrees as the corresponding plant polynomials). Furthermore, $\hat{M}(s)$ is stable, minimum phase and $k_m > 0$

13

Controller design: output feedback

- Controller structures
- Plant assumptions
- Controller design objective
- Matching equality



degree $n-2$ degree $n-1$

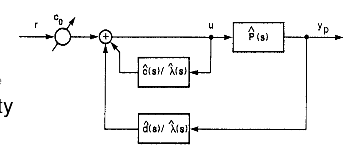
$$u = c_0 r + \frac{\hat{c}(s)}{\hat{\lambda}(s)}(u) + \frac{\hat{d}(s)}{\hat{\lambda}(s)}(y_p)$$

scalar degree $n-1$

14

Controller design: output feedback

- Controller structures
- Plant assumptions
- Controller design objective
- Matching equality



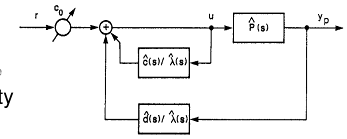
$$u = c_0 r + \frac{\hat{c}(s)}{\hat{\lambda}(s)}(u) + \frac{\hat{d}(s)}{\hat{\lambda}(s)}(y_p)$$

$$u = \frac{\hat{\lambda}}{\hat{\lambda}-\hat{c}}\left(c_0 r + \frac{\hat{d}}{\hat{\lambda}}(y_p)\right)$$

15

Controller design: output feedback

- Controller structures
- Plant assumptions
- Controller design objective
- Matching equality



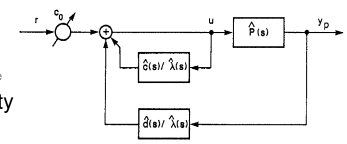
$$u = c_0 r + \frac{\hat{c}(s)}{\hat{\lambda}(s)}(u) + \frac{\hat{d}(s)}{\hat{\lambda}(s)}(y_p)$$

$$u = \frac{\hat{\lambda}}{\hat{\lambda}-\hat{c}}\left(c_0 r + \frac{\hat{d}}{\hat{\lambda}}(y_p)\right)$$

$$\frac{\hat{y}_p}{\hat{r}} = \frac{c_0 k_p \hat{\lambda} \hat{n}_p}{(\hat{\lambda}-\hat{c})\hat{d}_p - k_p \hat{n}_p \hat{d}}$$

Controller design: output feedback

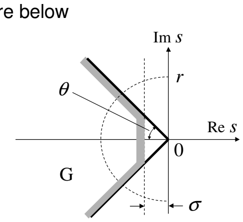
- Controller structures
- Plant assumptions
- Controller design objective
- Matching equality



Theorem: There exist unique $c_0^*, \hat{c}^*(s), \hat{d}^*(s)$ such that the transfer function from $r \to y_p$ is $\hat{M}(s)$.

Controller design: state feedback

- A rough guide
  - We may place all eigenvalues inside the region denoted by G in the figure below
  - It is better to place all eigenvalues evenly around a circle with radius r inside the sector as shown
  - A final selection may involve compromises among many conflicting requirements



Controller design: state feedback

- A rough guide
- Definition of state feedback

$$\dot{x} = Ax + bu, \quad y = cx$$

$$\downarrow \; u = r - kx$$

$$\dot{x} = (A - bk)x + br, \quad y = cx$$

19

Controller design: state feedback

- A rough guide
- Definition of state feedback
- Pole placement

Theorem: If $(A, \ b)$ is controllable, then by state feedback $u = r - kx$, where k is a 1 by n real constant vector, the eigenvalues of $A - bk$ can arbitrarily be assigned provided that complex conjugate eigenvalues are assigned in pairs.

20

Controller design: state feedback

- A rough guide
- Definition of state feedback
- Pole placement

Theorem (multivariable case)**:** All eigenvalues of $A - BK$ can be assigned arbitrarily (provided complex conjugate eigenvalues are assigned in pairs) by selecting a real constant matrix K if and only if $(A, \ B)$ is controllable.

21

Controller design: state feedback

- A rough guide
- Definition of state feedback
- Pole placement

Exercise: Given

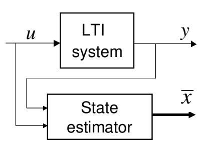
$$\dot{x} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u, \quad y = [1 \quad 1] x$$

find the control law $u = r - [k_1 \quad k_2] x$ (i.e., determine the values of k_1 and k_2) that places the closed-loop poles to be -1 and -2.

22

Controller design: state feedback

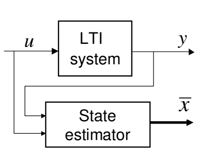
- A rough guide
- Definition of state feedback
- Pole placement
- State estimator

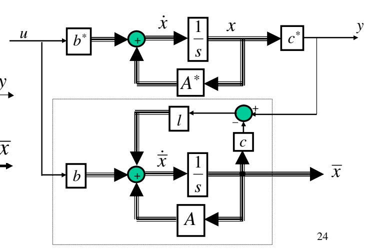


23

Controller design: state feedback

- A rough guide
- Definition of state feedback
- Pole placement
- State estimator

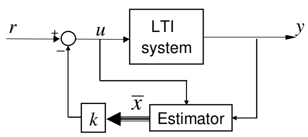




24

Controller design: state feedback

- A rough guide
- Definition of state feedback
- Pole placement
- State estimator
- Feedback from estimated states



An example

Control of an unknown mass: Consider the control of a mass on a frictionless surface by a motor force u, with the plant dynamics being $m\ddot{x} = u.$ Assume that a human operator provides the positioning command $r(t)$ to the control system (possibly through a joystick). A reasonable way of specifying the ideal response of the controlled mass to the external command $r(t)$ is to use the following reference model

$$\ddot{x}_m + \lambda_1 \dot{x}_m + \lambda_2 x_m = \lambda_2 r(t)$$

with positive constants λ_1 and λ_2 chosen to reflect the performance specifications. Design MRAC and STC for this system respectively.

References

- C.-T. Chen. Linear System Theory and Design, 3rd Edition, Oxford University Press, 1999.
- P. Ioannou and B. Fidan, Adaptive Control Tutorial, SIAM, 2006.
- S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence, and Robustness, Prentice-Hall, 1989.
- J.-J. E. Slotine and W. Li, Applied Nonlinear Control, Prentice Hall, 1991.