ECE 2646: Linear System Theory (3 Credits, Fall 2009)

Lecture 1: Course Organization and Introduction; Review of Classical Control; Review of Linear Algebra (I)

September 2, 2009

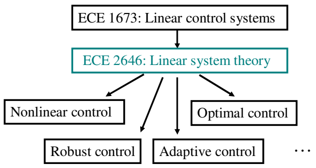
Instructor: Zhi-Hong Mao
Assistant Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

Outline

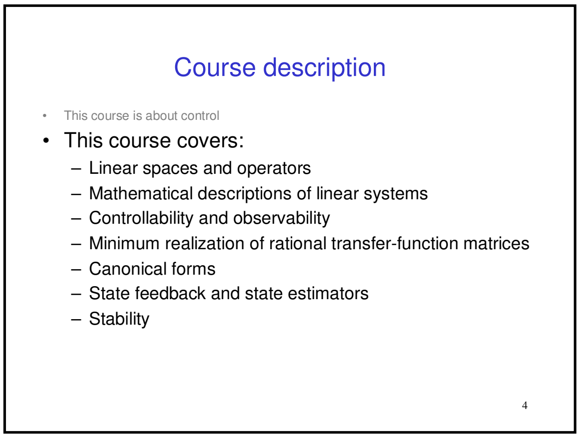
- Course description
- Course organization
- Why should we study control and study this course?
- Important concepts from classical control
- Features of this course
- Review of linear algebra (I)

Course description

- This course is about control
  - Transition from classical control (poles, zeros, transfer functions, etc., covered by ECE 1673) to modern control by introducing the notion of state space



Question: How many of you have taken ECE 1673 or a similar course on principles of automatic control?

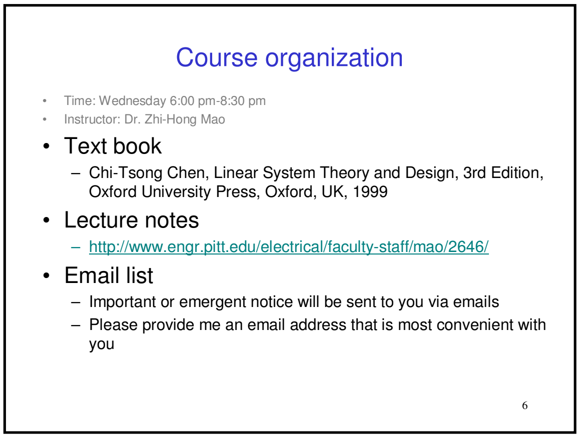


Course description

- This course is about control
- This course covers:
  - Linear spaces and operators
  - Mathematical descriptions of linear systems
  - Controllability and observability
  - Minimum realization of rational transfer-function matrices
  - Canonical forms
  - State feedback and state estimators
  - Stability

Course organization

- Time: Wednesday 6:00 pm–8:30 pm
- Instructor: Dr. Zhi-Hong Mao
  - (Office) 434 Benedum Hall
  - (Email) maozh@engr.pitt.edu
  - (Phone) 412-624-9674
  - (Office hours) Tuesday 4 pm–6 pm; visits at other times are also welcome



Course organization

- Time: Wednesday 6:00 pm-8:30 pm
- Instructor: Dr. Zhi-Hong Mao
- Text book
  - Chi-Tsong Chen, Linear System Theory and Design, 3rd Edition, Oxford University Press, Oxford, UK, 1999
- Lecture notes
  - http://www.engr.pitt.edu/electrical/faculty-staff/mao/2646/
- Email list
  - Important or emergent notice will be sent to you via emails
  - Please provide me an email address that is most convenient with you

Course organization

- Time: Wednesday 6:00 pm-8:30 pm
- Instructor: Dr. Zhi-Hong Mao
- Text book
- Lecture notes
- Email list
- Course evaluation
  - Homework and class participation: 35% (late homework will not be accepted)
  - Midterm: 30%
  - Final exam: 35%

Why should we know control and study this course?

- Practically all engineers will use control
- Control is an essential element of almost all engineering systems
  - It happens very often that systems perform poorly because they are designed from purely static analysis, with no consideration of dynamics and control

Why should we know control and study this course?

- Practically all engineers will use control
- Control is an essential element of almost all engineering systems
- Control can give designers extra degrees of freedom
- Control is not confined to engineering

Why should we know control and study this course?

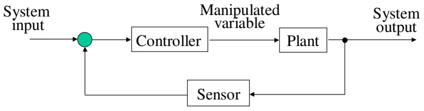
- Practically all engineers will use control
- Control is an essential element of almost all engineering systems
- Control can give designers extra degrees of freedom
- Control is not confined to engineering
- We need to know modern control as well as classical control

Why should we know control and study this course?

- Practically all engineers will use control
- Control is an essential element of almost all engineering systems
- Control can give designers extra degrees of freedom
- Control is not confined to engineering
- We need to know both modern and classical control
- This course is a good way to practice math in a genuine engineering contest
  - It involves extensive use of linear algebra
  - It is an excellent example showing that math is beautifully used in engineering applications
  - Word of caution: It may be boring if you do not like math

Important concepts from classical control

- What is control system?
  - The term control has many meanings and often varies between communities (e.g. “control group” in biomedical research)
  - Generally speaking, a control system is a system that is used to realize a desired output or objective
  - Most control in engineering systems uses feedback

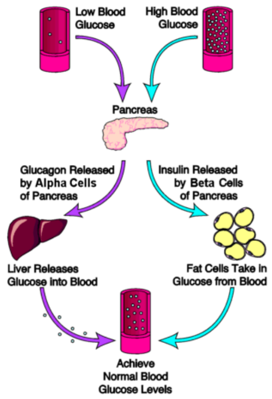


Important concepts from classical control

- What is control system?
- What is feedback?
  - The term feedback is used to refer to a situation where two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled
  - The principle of feedback is very simple: Base correcting actions on the difference between desired and actual performance

Question: Can you give us some examples of feedback control systems?

An example of feedback control in biological systems: normal regulation of blood glucose



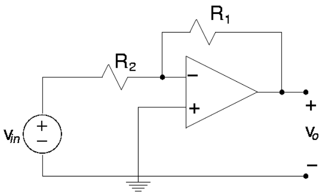
Important concepts from classical control

- What is control system?
- What is feedback?
  - The term feedback is used to refer to a situation where two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled
  - The principle of feedback is very simple: Base correcting actions on the difference between desired and actual performance
  - Advantages of using feedback
    - Reduction of steady-state error
    - Robustness to uncertainty
    - Higher levels of automation
    - Design of dynamics (e.g. stabilizing and linearizing)

NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC91-491-6 Date: September 13, 1991

X-29 at High Angle of Attack

X-29 (with forward swept wings) is unstable without control



Using nonlinear operational amplifier to realize linear operations

Important concepts from classical control

- What is control system?
- What is feedback?
  - The term feedback is used to refer to a situation where two (or more) dynamical systems are connected together such that each system influences the other and their dynamics are thus strongly coupled
  - The principle of feedback is very simple: Base correcting actions on the difference between desired and actual performance
  - Advantages of using feedback
  - Drawbacks of feedback
    - Feedback may introduce instability if not properly designed (e.g., the effects of “positive feedback” when the amplification on a microphone is turned up too high in a room)
    - Feedback inherently couples different parts of a system (injecting measurement noise into the system)
    - Feedback increases system complexity (embedding a control system into a product)

Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
  - Refer to a system whose behavior changes over time, often in response to external stimulation or forcing
  - Good control must take into account the dynamic behavior of the object being controlled in order to do a good job
  - Example: Ballistic movement of reaching

Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?

$$u(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

Models in classical control ≈
differential equations of dynamical systems

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t)$$

Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?

$$u(t) \longrightarrow \boxed{\text{System}} \longrightarrow y(t)$$

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \cdots + b_1 \frac{du(t)}{dt} + b_0 u(t)$$

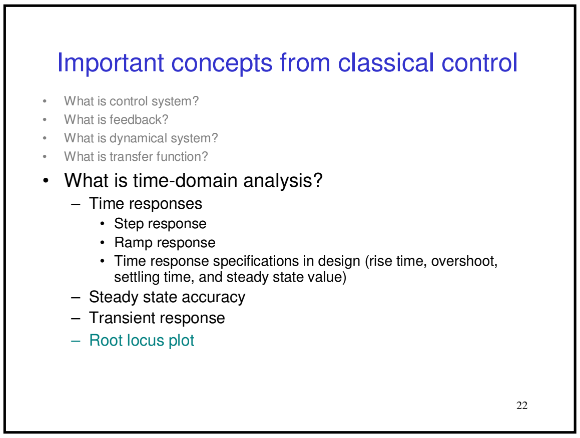
Laplace transform
(with zero initial conditions)

$$\left(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right)Y(s) = \left(b_m s^m + \cdots + b_1 s + b_0\right)U(s)$$

$$\frac{Y(s)}{U(s)} \equiv G(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

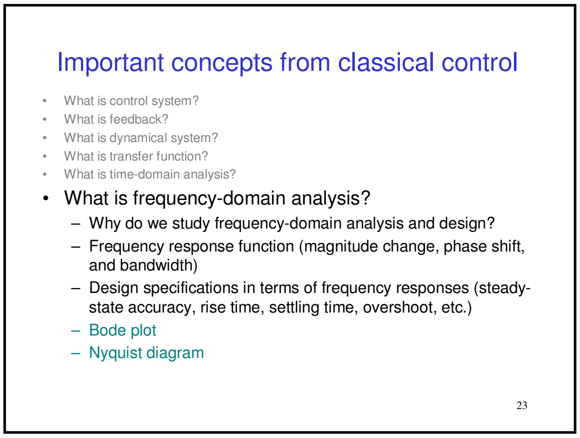
$G(s)$ is called a transfer function

$$U(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$



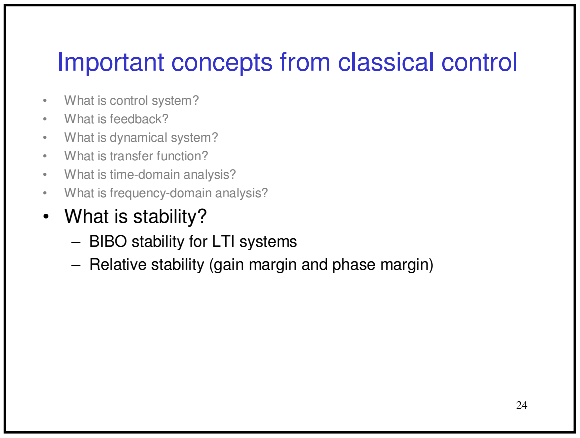
Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?
- What is time-domain analysis?
  - Time responses
    - Step response
    - Ramp response
    - Time response specifications in design (rise time, overshoot, settling time, and steady state value)
  - Steady state accuracy
  - Transient response
  - Root locus plot



Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?
- What is time-domain analysis?
- What is frequency-domain analysis?
  - Why do we study frequency-domain analysis and design?
  - Frequency response function (magnitude change, phase shift, and bandwidth)
  - Design specifications in terms of frequency responses (steady-state accuracy, rise time, settling time, overshoot, etc.)
  - Bode plot
  - Nyquist diagram



Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?
- What is time-domain analysis?
- What is frequency-domain analysis?
- What is stability?
  - BIBO stability for LTI systems
  - Relative stability (gain margin and phase margin)

Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?
- What is time-domain analysis?
- What is frequency-domain analysis?
- What is stability?
- What is sensitivity?
  - Disturbance rejection
  - Sensitivity and stability margin may exert conflicting requirements in control design

Important concepts from classical control

- What is control system?
- What is feedback?
- What is dynamical system?
- What is transfer function?
- What is time-domain analysis?
- What is frequency-domain analysis?
- What is stability?
- What is sensitivity?
- What are the commonly used controllers?
  - Gain compensator (proportional controller)
  - Phase-lead compensator
  - Phase-lag compensator
  - Lag-lead compensator
  - PID controller

Features of this course

- Use state-space description
  - The system is modeled as a set of first-order differential equations (representation of the dynamics of an n th-order system using n first-order differential equations)

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t)$$
$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t)$$

Features of this course

- Use state-space description
  - The system is modeled as a set of first-order differential equations
  - Example: Newton's Second Law

$$m\frac{d^2y(t)}{dt^2}=u(t) \longrightarrow \begin{aligned} x_1(t)&=y(t)\\ \frac{dx_1(t)}{dt}&=x_2(t)\\ \frac{dx_2(t)}{dt}&=\frac{u(t)}{m}\end{aligned}$$

$$\Big\downarrow$$

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\ 1/m\end{bmatrix}u(t)$$

$$y(t)=[1\ \ 0]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

$$\boxed{\begin{aligned}\dot{\mathbf{x}}(t)&=\mathbf{A}(t)\mathbf{x}(t)+\mathbf{B}(t)\mathbf{u}(t)\\ \mathbf{y}(t)&=\mathbf{C}(t)\mathbf{x}(t)+\mathbf{D}(t)\mathbf{u}(t)\end{aligned}}$$

Question: What are **A**, **B**, **C**, and **D** for this example?

Features of this course

- Use state-space description
  - The system is modeled as a set of first-order differential equations
  - Example: Newton's Second Law
  - Exercise: Please give a state-space description for the following system? Hint: Consider $x_1 = y,\ x_2 = \dot{y}$

$$\boxed{\ddot{y}+2\dot{y}-3y=u}$$

Question: Is the state-space description unique? Hint: Consider $x_1 = y,\ x_2 .= \dot{y}+2y$

Features of this course

- Use state-space description
  - The system is modeled as a set of first-order differential equations
  - The power of modern control has its roots in the fact that the state-space model can represent a MIMO (multi-input multi-output) system as a SISO (single-input single-output) system due to the use of vectors and matrices

Features of this course

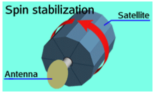
- Use state-space description
- Why use state-space approach?
  - State variable form convenient way to work with complex dynamics; matrix format easy to use on computers
  - Transfer functions only deal with input/output behavior, while state-space form provides easy access to the internal features and response of the system
  - State-space approach is great for MIMO (multi-input multi-output) system, which are very hard to work with using transfer functions

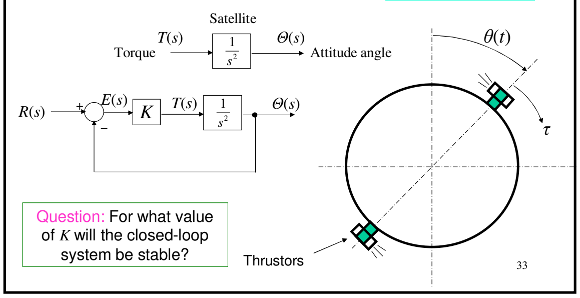
Features of this course

- Use state-space description
- Why use state-space approach?
  - State variable form convenient way to work with complex dynamics; matrix format easy to use on computers
  - Transfer functions only deal with input/output behavior, while state-space form provides easy access to the internal features and response of the system
  - State-space approach is great for MIMO (multi-input multi-output) system, which are very hard to work with using transfer functions
  - State variables can be used for feedback
    - Example: Attitude control for a rigid satellite

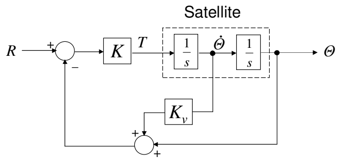
Attitude control for a rigid satellite

$$\tau(t) = J\frac{d^2\theta(t)}{dt^2}$$





Question: For what value of K will the closed-loop system be stable?



Attitude control for a rigid satellite with state variable feedback $(x_1 = \Theta,\ x_2 = \dot{\Theta})$

Features of this course

- Use state-space description
- Why use state-space approach?
- Concentrate on linear systems
  - Linear models describe small perturbations from nominal operations, and most control design is aimed at regulating such perturbations
  - Linear models are far more tractable than general nonlinear models, so systematic and detailed control design approaches can be developed

Features of this course

- Use state-space description
- Why use state-space approach?
- Concentrate on linear systems
- Apply time-domain techniques
  - The frequency-domain approach is at its best when dealing with SISO systems, for the graphical techniques were inconvenient to apply with multiple inputs and outputs
  - With the advent of the space age, control design turned away from the frequency-domain techniques of classical control theory and back to the differential equation techniques of the late 1800's, which were couched in the time domain

Features of this course

- Use state-space description
- Why use state-space approach?
- Concentrate on linear systems
- Apply time-domain techniques
  - The frequency-domain approach is at its best when dealing with SISO systems, for the graphical techniques were inconvenient to apply with multiple inputs and outputs
  - With the advent of the space age, control design turned away from the frequency-domain techniques of classical control theory and back to the differential equation techniques of the late 1800's, which were couched in the time domain
  - However, modern control that was based on time-domain techniques was lacking in some aspects
    - By solving matrix design equations, it is often possible to design a control system that works in theory without gaining any engineering intuition about the problem; on the other hand, the frequency-domain techniques of classical control theory impart a great deal of intuition

Features of this course

- Use state-space description
- Why use state-space approach?
- Concentrate on linear systems
- Apply time-domain techniques
  - The frequency-domain approach is at its best when dealing with SISO systems, for the graphical techniques were inconvenient to apply with multiple inputs and outputs
  - With the advent of the space age, control design turned away from the frequency-domain techniques of classical control theory and back to the differential equation techniques of the late 1800's, which were couched in the time domain
  - However, modern control that was based on time-domain techniques was lacking in some aspects
    - By solving matrix design equations, it is often possible to design a control system that works in theory without gaining any engineering intuition about the problem; on the other hand, the frequency-domain techniques of classical control theory impart a great deal of intuition
    - A modern control system with any compensator dynamics can fail to be robust to disturbances, unmodeled dynamics, and measurement noise; on the other hand, robustness is built in with a frequency-domain approach using notions like the gain and phase margin

Features of this course

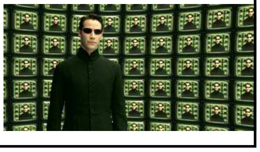
- Use state-space description
- Why use state-space approach?
- Concentrate on linear systems
- Apply time-domain techniques
  - The frequency-domain approach is at its best when dealing with SISO systems, for the graphical techniques were inconvenient to apply with multiple inputs and outputs
  - With the advent of the space age, control design turned away from the frequency-domain techniques of classical control theory and back to the differential equation techniques of the late 1800's, which were couched in the time domain
  - However, modern control that was based on time-domain techniques was lacking in some aspects
  - Nowadays, many of the classical frequency-domain techniques can be incorporated into time-domain design

Review of linear algebra (I)

- Linear algebra quiz
  - Problem 1: Given $a1=[0\ 1\ 2]'$, $a2=[1\ 2\ 0]'$, $a3=[1\ 3\ 2]'$, $a4=[2\ 4\ 0]'$, are they linearly dependent or independent?
  - Problem 2: What are the eigenvalues of matrix C?

$$C = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix} \qquad D = \begin{bmatrix} \lambda_1^2 & \lambda_2^2 & \lambda_3^2 \\ \lambda_1 & \lambda_2 & \lambda_3 \\ 1 & 1 & 1 \end{bmatrix}$$

  - Problem 3: What is the rank of matrix D?
  - Problem 4: What is this?



Review of linear algebra (I)

- Linear algebra quiz
- Important concepts
  - Vector and vector space (or linear space)
  - Matrix
  - Linearly dependent and linearly independent
  - Rank of a matrix
  - Basis and representation
  - Norm of vectors
  - Eigen values and eigen vectors
  - Diagonalization of a matrix