ECE 1673: Linear Control Systems
(4 Credits, Spring 2009)

Lecture 8: More on Stability Analysis; Root Locus Plot

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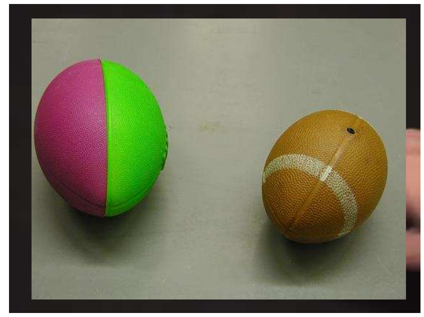
Spinning football: about rotational stability



About last lecture

- Steady-state error and system type

$$G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$$

N	$R(s)$: $1/s$	$1/s^2$	$1/s^3$	Error constants
0	$\frac{1}{1+K_p}$	∞	∞	$K_p = \lim_{s\to 0} G_c G_p$
1	0	$\frac{1}{K_v}$	∞	$K_v = \lim_{s\to 0} s G_c G_p$
2	0	0	$\frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2 G_c G_p$

About last lecture

- Steady-state error and system type
- More on integrator $1/s$
- Transient response

$$\text{Transient response} = k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_n e^{p_n t}$$

  - System modes
  - Effect of zeros

Outline of this lecture

- "Review" of Routh stability criterion
- Stability by simulation
- Introduction to root locus
- Root-locus example: pitch control of the 1903 Wright Flyer

Routh stability criterion

- Definition
  - The Routh criterion is a method for determining stability for systems with an nth-order characteristic equation of the form:

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0$$

Routh stability criterion

- Definition $a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0$
- Routh table

$$
\begin{array}{c|cccc}
s^n & a_n & a_{n-2} & a_{n-4} & \cdots \\
s^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & \cdots \\
s^{n-3} & c_1 & c_2 & c_3 & \cdots \\
\cdot & \cdot & \cdot & \cdot & \cdots
\end{array}
$$

$$b_1 = -\frac{1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{vmatrix}, \quad b_2 = -\frac{1}{a_{n-1}}\begin{vmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{vmatrix}, \cdots$$

$$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}, \quad c_2 = -\frac{1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{vmatrix}, \cdots$$

  - The determinant in the expression for the ith coefficient in a row is formed from the first column and the $(i+1)$th column of the two preceding rows
  - The table is continued horizontally and vertically until only zeros are obtained

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Routh stability criterion

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- Routh criterion
  - All the roots of the characteristic equation have negative real parts if and only if the elements of the first column of the Routh table have the same sign. Otherwise, the number of roots with positive real parts is equal to the number of changes of sign
  - Exercise:

$$Q(s) = (s+2)(s^2 - s + 4) = s^3 + s^2 + 2s + 8 = 0$$

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Routh stability criterion

- Definition
- Routh table
- Routh criterion
- More on characteristic polynomial

$$Q_n(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

  - In any coefficient a_i is equal to zero, then not all the roots are in the left half-plane
  - If any coefficient a_i is negative, then at least one root is in the right half-plane
  - Exercises: $s^3 + 2s^2 + 3s - 1, \quad s^4 + s^2 + 1$

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Stability by simulation

- Physical systems are generally not LTI
- For a complex nonlinear system, a simulation may be the only method available for determining the characteristics (including stability) of the system

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Stability by simulation

- Physical systems are generally not LTI
- For a complex nonlinear system, a simulation may be the only method available for determining the characteristics (including stability) of the system
- An example: M2 design

* August 14, 1998: M2 configuration decided
* August 28, 1998: Planar simulation with M2 weight parameters walking
* September 4, 1998: Preliminary CAD assembly of M2 done
* September 8, 1998: M2 Weight budget determined
* September 8, 1998: First progress meeting
* October 3, 1998: Full scale mockup of M2 built
* December 11, 1998: Three dimensional simulation of M2 walking
* December 30, 1998: Frameless actuator prototype (FRAP) built
* January 14, 1999: Frameless actuator prototype tested
* January 15, 1999: Mechanical power requirements determined
* February 15, 1999: Foot designed
* April 16, 1999: Final actuator designed
* May 19, 1999: 12 week lead parts ordered
* June 10, 1999: M2 leg designed
* June 30, 1999: M2 Vestibular system designed
* July 30, 1999: Electronics designed
* October 25, 1999: Full CAD assembly model completed
* November 30, 1999: Machined parts sent out
* November 30, 1999: Electronic Boards and Parts Ordered
* December 16, 1999: Begin M2 Assembly
* January 6, 2000: Force control boards populated
* March 2000: Mechanical assembly complete
* April, 2000: Complete system integration
* April 15, 2000: M2 standing
* July, 2000: M2 takes a step
* August, 2000: M2 steps in place

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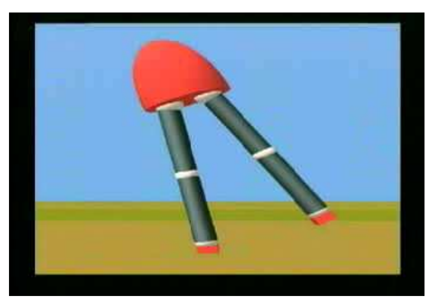

Three dimensional simulation of M2

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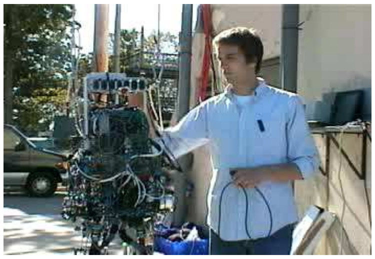


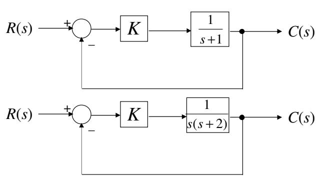
Real M2 ...

Introduction to root locus

- Definition
  - A root locus of a system is a plot of the roots of the system characteristic equation (the poles of the closed-loop transfer function) as some parameter of the system is varied

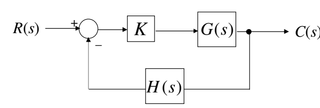
Introduction to root locus

- Definition
  - A root locus of a system is a plot of the roots of the system characteristic equation (the poles of the closed-loop transfer function) as some parameter of the system is varied
  - Exercises: What are the root loci of the following systems?



Introduction to root locus

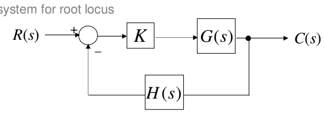
- Definition
- General system for root locus



  - $G(s)$ includes both the compensator transfer function $G_c(s)$ and plant transfer function $G_p(s)$
  - Only consider nonnegative K

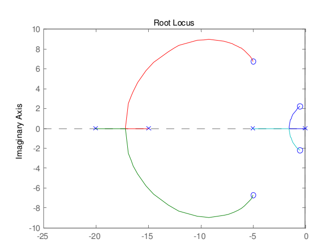
Introduction to root locus

- Definition
- General system for root locus



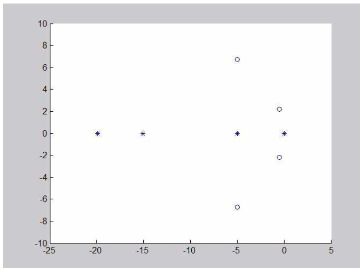
- Matlab program for root-locus plot: `rlocus`
  - An example: $G(s) = \frac{(s^2+s+5)(s^2+10s+70)}{s(s+5)(s+15)(s+20)}, H(s) = 1$

```
num = conv([1 1 5], [1 10 70]);
den = conv(conv([1 0], [1 5]),conv([1 15],[1 20]));
rlocus(num,den);
```

Any observations?

Example 1



Any observations?

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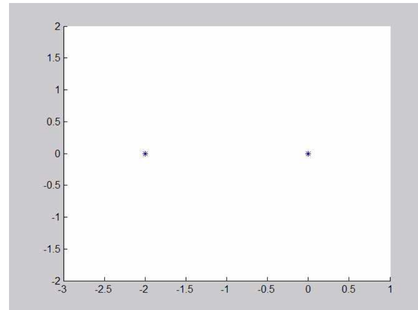
Example 2



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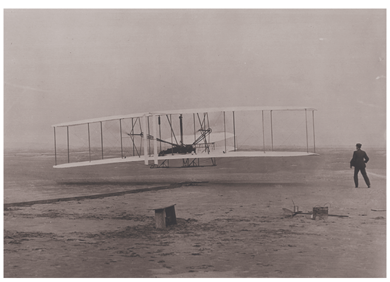
Example: pitch control of the 1903 Wright Flyer



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Example: pitch control of the 1903 Wright Flyer

- What is pitch control?

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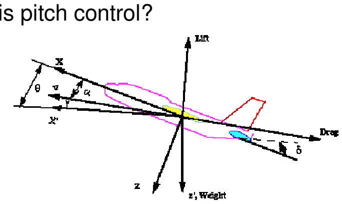
Example: pitch control of the 1903 Wright Flyer

- What is pitch control?



The pitch angle of an airplane is controlled by adjusting the angle (and therefore the lift force) of the rear elevator. The aerodynamic forces (lift and drag) as well as the airplane's inertia are taken into account.

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Example: pitch control of the 1903 Wright Flyer

- What is pitch control?
  - "Cobra"

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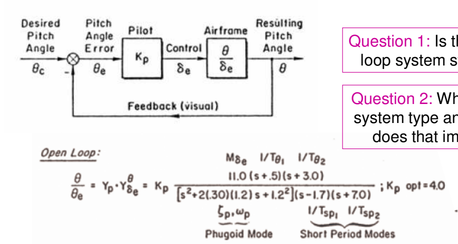
Example: pitch control of the 1903 Wright Flyer

- What is pitch control?
- Pitch control system of 1903 Wright Flyer

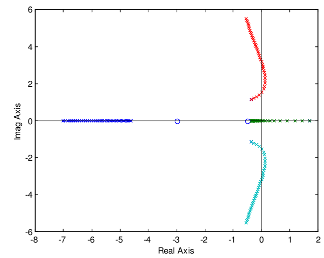


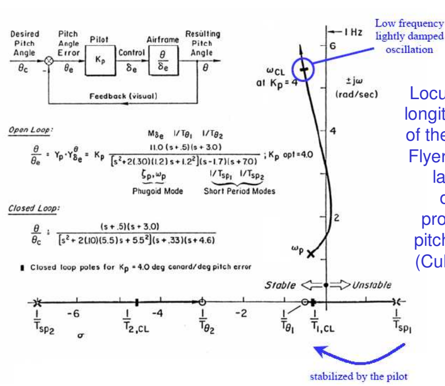
Question 1: Is the open loop system stable?

Question 2: What is its system type and what does that imply?

Example: pitch control of the 1903 Wright Flyer

- What is pitch control?
- Pitch control system of 1903 Wright Flyer
  – Root-locus plot





Locus of roots for longitudinal motion of the 1903 Wright Flyer; pilot control law: canard deflection proportional to pitch angle error (Culick and Jex, 1984)

References

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- http://aeroweb.lucia.it/rap/RAFAQ/cobra.html
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