ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 7: Steady-State Accuracy, System Types, and Transient Response

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About Homework 3 and Lab 2

- Homework 3
  - Problems 5.14, 6.5, and 6.8 (a)-(c) in the text book
  - Due 2/4 Wednesday
- Lab 2
  - Due 2/11 Wednesday

About last lecture

- Sensitivity

$$S_b^W = \frac{\partial W}{\partial b}\frac{b}{W}$$

Question 1: How to reduce the sensitivity of the closed-loop system characteristics to the parameters within the plant?

$$S_{G_p}^T = \frac{1}{1+G_cG_pH}$$

Question 2: How to reduce the sensitivity of the closed-loop system characteristics to the parameters within the sensor?

$$S_H^T = \frac{-G_cG_pH}{1+G_cG_pH}$$

Question 3: How to reduce the sensitivity of the open-loop system characteristics to the parameters within the plant?

About last lecture

- Sensitivity
- Disturbance rejection

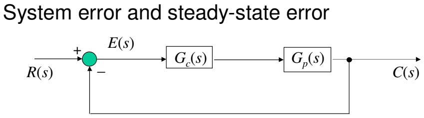
Question: How to reject disturbances?

Outline of this lecture

- Steady-state accuracy and system type
- More on integrator $1/s$
- Transient response

Steady-state accuracy

- System error and steady-state error



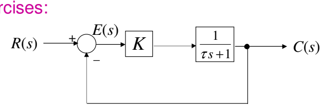
$$C(s) = \frac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)}R(s)$$

$$E(s) = \frac{?}{??}R(s)$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{sR(s)}{1+G_c(s)G_p(s)}$$

Steady-state accuracy

- System error and steady-state error
  - Exercises:



Question 1: For unit step input, what is e_{ss}?

Question 2: For unit ramp input, what is e_{ss}?

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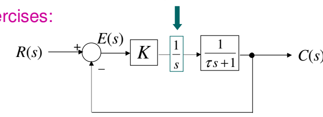
Steady-state accuracy

- System error and steady-state error
  - Exercises:



Question 1: For unit step input, what is e_{ss}?

Question 2: For unit ramp input, what is e_{ss}?

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Steady-state accuracy

- System error and steady-state error
- System type

$$G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$$

  - N is called the system type

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Steady-state accuracy

- System error and steady-state error
- System type $G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$
- Position error constant K_p (for step response)

$$e_{ss} = \lim_{s\to 0} \frac{1}{1+G_c(s)G_p(s)} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s\to 0} G_c(s)G_p(s)$$

  - For the case that N is greater than or equal to 1, K_p is unbounded and the steady-state error for a step input is zero

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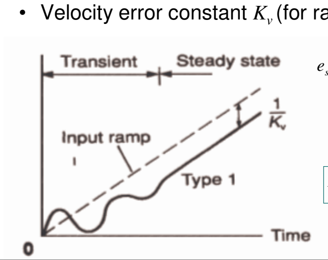
Steady-state accuracy

- System error and steady-state error
- System type $G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$
- Position error constant K_p (for step response)
- Velocity error constant K_v (for ramp response)



$$e_{ss} = \lim_{s\to 0} \frac{1/s}{1+G_c(s)G_p(s)}$$

$$= \lim_{s\to 0} \frac{1}{sG_c(s)G_p(s)} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} sG_c(s)G_p(s)$$

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Steady-state accuracy

- System error and steady-state error
- System type $G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$
- Position error constant K_p (for step response)
- Velocity error constant K_v (for ramp response)

$$e_{ss} = \lim_{s\to 0} \frac{1/s}{1+G_c(s)G_p(s)} = \lim_{s\to 0} \frac{1}{sG_c(s)G_p(s)} = \frac{1}{K_v}$$

$$K_v = \lim_{s\to 0} sG_c(s)G_p(s)$$

  - For a type 2 and higher system, the steady-state error for a ramp input is zero

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Steady-state accuracy

- System error and steady-state error
- System type $G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$
- Position error constant K_p (for step response)
- Velocity error constant K_v (for ramp response)
- Acceleration error constant K_a (for parabolic input)

$$e_{ss} = \lim_{s\to 0} \frac{1/s^2}{1+G_c(s)G_p(s)} = \lim_{s\to 0} \frac{1}{s^2 G_c(s)G_p(s)} = \frac{1}{K_a}$$

$$K_a = \lim_{s\to 0} s^2 G_c(s)G_p(s)$$

  – For a type 3 and higher system, the steady-state error for a parabolic input is zero

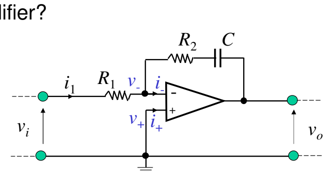
Steady-state accuracy

- System error and steady-state error
- System type $G_c(s)G_p(s) = \frac{F(s)}{s^N Q_1(s)}$
- Position error constant K_p (for step response)
- Velocity error constant K_v (for ramp response)
- Acceleration error constant K_a (for parabolic input)

N	$R(s)$ $1/s$	$1/s^2$	$1/s^3$	Error constants
0	$\frac{1}{1+K_p}$	∞	∞	$K_p = \lim_{s\to 0} G_c G_p$
1	0	$\frac{1}{K_v}$	∞	$K_v = \lim_{s\to 0} s G_c G_p$
2	0	0	$\frac{1}{K_a}$	$K_a = \lim_{s\to 0} s^2 G_c G_p$

More on integrator $1/s$

- How to realize an integrator with an operational amplifier?



Assumptions: (1) Input impedance is infinite, i.e., $i_- = i_+ = 0$, (2) $v_- = v_+$, (3) the output impedance is zero

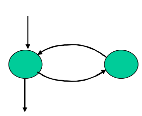
Question: What is the transfer function from v_i to v_o (what if $R_2 = 0$)?

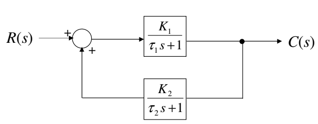
More on integrator $1/s$

- How to realize an integrator with an operational amplifier?
- Evidence for the existence of "integrators" in the brain
  – Electrical stimulation in the motor cortex of monkeys
  – Working memory
  – How the brain keeps the eyes still?

More on integrator $1/s$

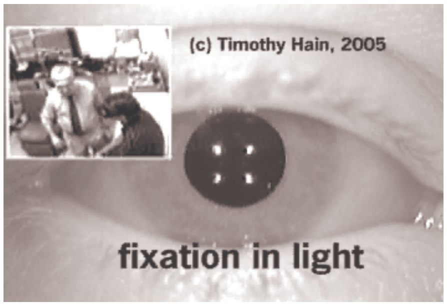
- How to realize an integrator with an operational amplifier?
- Evidence for the existence of "integrators" in the brain
- How to realize integrating circuit with neural networks?
  – Leaky integrators
  – Reverberatory connections (using positive feedback)





More on integrator $1/s$

- How to realize an integrator with an operational amplifier?
- Evidence for the existence of "integrators" in the brain
- How to realize integrating circuit with neural networks?
- An example of dysfunction of integrators in the brain: nystagmus



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Transient response

$$R(s) \longrightarrow \boxed{T(s)} \longrightarrow C(s)$$

$$T(s) = \frac{P(s)}{Q(s)}, \quad Q(s) = a_n \prod_{i=1}^{n} (s - p_i)$$

$$C(s) = T(s)R(s) = \frac{P(s)}{a_n \prod_{i=1}^{n} (s - p_i)} R(s) = \frac{k_1}{s - p_1} + \cdots + \frac{k_n}{s - p_n} + C_r(s)$$

$$\text{Transient response} = k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_n e^{p_n t}$$

- Modes of the system

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Transient response

$$\text{Transient response} = k_1 e^{p_1 t} + k_2 e^{p_2 t} + \cdots + k_n e^{p_n t}$$

- Modes of the system
- Dominant pole(s)
- Effect of zeros
  - A zero near a pole may reduce the effect of the pole

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References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- http://www.dizziness-and-balance.com/practice/gen.htm

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