ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 5: System Responses (Cont'd) and Stability

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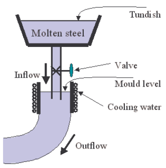
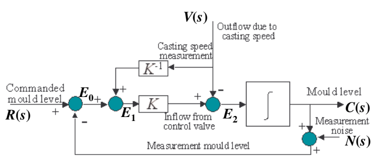
Outline of this lecture

- An exercise from Lab 1
- Review of Lecture 4
- System responses
- Stability

An exercise from Lab 1





- What is the relationship between the output $C(s)$ and the three inputs $R(s)$, $V(s)$, and $N(s)$?

Review of Lecture 4

- What are the forced response, steady state response, natural response, transient response in the step response of a first-order system?

$$G(s) = \frac{C(s)}{R(s)} = \frac{K}{\tau s + 1}$$

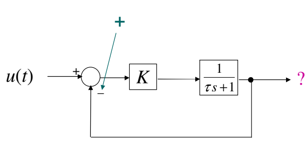
$$R(s) = 1/s,$$

$$C(s) = \frac{1}{s}\frac{K/\tau}{s+1/\tau} = \frac{K}{s} - \frac{K}{s+1/\tau},$$

$$c(t) = K - Ke^{-t/\tau}, \quad t > 0$$

Review of Lecture 4

- What if we use positive feedback in the following closed-loop system?



Review of Lecture 4

- What is steady-state gain (or dc gain) of a system?
- Given $G(s)$, the transfer function of a system, how to calculate its steady-state gain?

> Remark: The system dc gain is the steady-state gain to a constant input for the case the output has a final value, and it is equal to the system transfer function evaluated at $s = 0$.

Review of Lecture 4

- Second-order systems

$$G(s)=\frac{C(s)}{R(s)}=\frac{b_0}{s^2+a_1 s+a_0}=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

? ?

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Review of Lecture 4

$$G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Is this the direction of increase or decrease of ζ ?

- Step response

Case 1: $\zeta<1$ (underdamped)

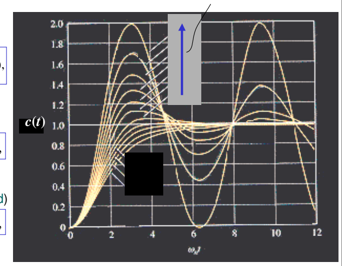
$$c(t)=1-\frac{1}{\beta}e^{-\zeta\omega_n t}\sin(\beta\omega_n t+\theta),$$

Case 2: $\zeta>1$ (overdamped)

$$c(t)=1+k_1e^{-t/\tau_1}+k_2e^{-t/\tau_2},$$

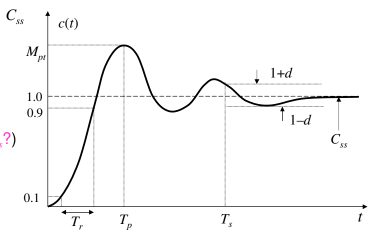
Case 3: $\zeta=1$ (critically damped)

$$c(t)=1+k_1e^{-t/\tau}+k_2te^{-t/\tau},$$



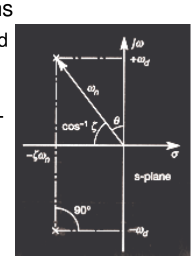
Time response of specifications in design

- Some parameters
  - Rise time, T_r
  - Peak value of the step response, M_{pt} ; time to reach it, T_p (how to calculate T_p?)
  - Steady state value, C_{ss}
  - Percent overshoot, $\frac{M_{pt}-C_{ss}}{C_{ss}}\times 100$
  - Settling time, T_s (how to calculate T_s?)



Time response of specifications in design

- Some parameters
- Time response and pole locations
  - The settling time is inversely related to the real part of the poles (the speed of response is increased by moving the poles to the left in the s-plane) $T_s=k\tau=\frac{k}{\zeta\omega_n}$
  - Decreasing the angle $\cos^{-1}\zeta$ (increasing ζ) reduces the percent overshoot



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Time response of specifications in design

- Some parameters
- Time response and pole locations
  - The settling time is inversely related to the real part of the poles (the speed of response is increased by moving the poles to the left in the s-plane)
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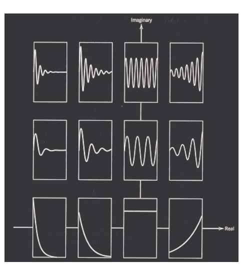

This picture shows how changing pole locations in the s-plane affects responses

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Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs

$$r(t)=A\cos\omega_1 t,\quad R(s)=\frac{As}{s^2+\omega_1^2},\qquad \text{Assume that } \lim_{t\to\infty}c_g(t)=0$$

$$C(s)=G(s)R(s)=\frac{k_1}{s-j\omega_1}+\frac{k_2}{s+j\omega_1}+C_g(s)$$

$$k_1=\frac{1}{2}AG(j\omega_1),\ k_2=\frac{1}{2}AG(-j\omega_1),\quad G(j\omega_1)=|G(j\omega_1)|e^{j\phi(\omega_1)}$$

$$c_{ss}(t)=k_1e^{j\omega_1 t}+k_2e^{-j\omega_1 t}=A|G(j\omega_1)|\frac{e^{j(\omega_1 t+\phi(\omega_1))}+e^{-j(\omega_1 t+\phi(\omega_1))}}{2}$$

$$=A|G(j\omega_1)|\cos(\omega_1 t+\phi(\omega_1))$$

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Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs

$$r(t) = A\cos\omega_1 t, \quad G(j\omega_1) = |G(j\omega_1)|e^{j\phi(\omega_1)}$$

$$c_{ss}(t) = A|G(j\omega_1)|\cos(\omega_1 t + \phi(\omega_1))$$

  – The steady-state gain of a system for a sinusoidal input is the magnitude of the transfer function evaluation at $s = j\omega_1$, and the phase shift of the output sinusoid relative to the input sinusoid is the angle of $G(j\omega_1)$

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Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
  – The steady-state gain of a system for a sinusoidal input is the *magnitude* of the transfer function evaluation at $s = j\omega_1$, and the *phase shift* of the output sinusoid relative to the input sinusoid is the angle of $G(j\omega_1)$
  – $G(j\omega)$ is defined as the frequency response function

$$G(j\omega) = |G(j\omega)|e^{j\phi(\omega)}$$

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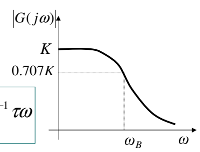
Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response of first-order systems

$$G(s) = \frac{K}{\tau s + 1}$$



$$|G(j\omega)| = \frac{K}{(1+\tau^2\omega^2)^{1/2}}, \quad \phi(\omega) = -\tan^{-1}\tau\omega$$

  – System bandwidth, ω_B: The frequency at which the gain is equal to 1/sqrt(2) (approximately 0.707) times the gain at very low frequencies

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Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response of first-order systems
- Frequency response of second-order systems

$$G(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2} = \frac{1}{(s/\omega_n)^2 + 2\varsigma(s/\omega_n) + 1}$$

$$G(j\omega) = \frac{1}{[1-(\omega/\omega_n)^2] + j2\varsigma(\omega/\omega_n)}$$

$$|G(j\omega)| = \frac{1}{\left[\left(1-(\omega/\omega_n)^2\right)^2 + \left(2\varsigma(\omega/\omega_n)\right)^2\right]^{\frac{1}{2}}}$$

Question: What will happen if $\zeta = 0$ and $\omega = \omega_n$?

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Frequency response of systems

- Frequency response: steady-state response of systems to sinusoidal inputs
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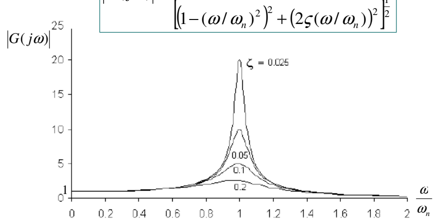
$$|G(j\omega)| = \frac{1}{\left[\left(1-(\omega/\omega_n)^2\right)^2 + \left(2\varsigma(\omega/\omega_n)\right)^2\right]^{\frac{1}{2}}}$$



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Stability

- What do we usually mean by stability?
  – Examples of stable and unstable systems

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M2-F2 experiencing lateral oscillations in flight

[1] [2]

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Rest tremor in Parkinson's disease

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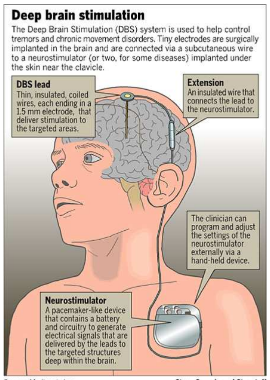

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Deep brain stimulation for the treatment of rest tremor in Parkinson's disease

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Stability

- What do we usually mean by stability?
- Bounded-input, bounded-output (BIBO) stability
  - A system is BIBO stable, if, for every bounded input, the output remains bounded for all time
  - The concept of BIBO stability is not appropriate to describe stability in all nonlinear systems

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Stability

- What do we usually mean by stability?
- Bounded-input, bounded-output (BIBO) stability
- Criteria for the BIBO stability of linear time-invariant systems

$$R(s) \longrightarrow \boxed{T(s)} \longrightarrow C(s) \qquad T(s)=\frac{P(s)}{Q(s)}, \quad Q(s)=a_n\prod_{i=1}^{n}(s-p_i)$$

$$C(s)=T(s)R(s)=\frac{P(s)}{a_n\prod_{i=1}^{n}(s-p_i)}R(s)=\frac{k_1}{s-p_1}+\cdots+\frac{k_n}{s-p_n}+C_r(s)$$

  - Some terms
    - Characteristic equation: $Q(s)=0$
    - System roots or system poles: $p_i,\ i=1,\ldots,n$
    - Forced response: $C_r(s)$ (the sum of terms, in the partial-fraction expansion, that originate in the poles of $R(s)$)

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Stability

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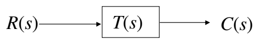
- Criteria for the BIBO stability of linear time-invariant systems



$$T(s)=\frac{P(s)}{Q(s)},\quad Q(s)=a_n\prod_{i=1}^{n}(s-p_i)$$

$$C(s)=T(s)R(s)=\frac{P(s)}{a_n\prod_{i=1}^{n}(s-p_i)}R(s)=\frac{k_1}{s-p_1}+\cdots+\frac{k_n}{s-p_n}+C_r(s)$$

  - Some terms
  - A linear time-invariant system is BIBO stable provided all roots of the system characteristic equation (poles of the closed-loop transfer function) lie in the left half of the s-plane (why?)

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References

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