ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 3: System Modeling, Block Diagrams, and Signal Flow Graphs

January 12, 2009

Instructor: Zhi-Hong Mao
Assistant Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

Review of last lecture

- Complex variables
  - Definition of poles and zeros

Question: What are the zeros and poles of the following functions?

$$\frac{(s+2)}{s(s+1)(s^2+3)}, \quad e^{5s}$$

Review of last lecture

- Complex variables
- Differential equations
  - Linearization of nonlinear differential equations
  - Solving linear ODE with classical method and Laplace transform

Review of last lecture

- Complex variables
- Differential equations
- Laplace transform
  - Partial fraction expansion of a rational function

Exercise: Find partial fraction expansions of the following functions:

$$\frac{(s+2)}{s(s+1)(s+3)^2}, \quad \frac{1}{s(s^2+2s+5)}$$

Review of last lecture

- Complex variables
- Differential equations
- Laplace transform
  - Partial fraction expansion of a rational function
  - Final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

The final-value theorem is valid only if $sF(s)$ does not have any poles on the $j\omega$ axis and in the right half of the s-plane.

Examples:

$$F_1(s) = \frac{5}{s(s^2+s+2)}, \quad F_2(s) = \frac{\omega}{s^2+\omega^2}$$

Review of last lecture

- Complex variables
- Differential equations
- Laplace transform
  - Partial fraction expansion of a rational function
  - Final value theorem
  - Differential theorem

$$L\left[\frac{df}{dt}\right] = sF(s) - f(0^-),$$

$$L\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - s^{n-1} f(0^-) - \cdots - f^{n-1}(0^-),$$

$$\text{where } f(0^-) = \lim_{t\to 0} f(t), \quad t < 0.$$

Note that in the text book

$$L\left[\frac{df}{dt}\right] = sF(s) - f(0^+),$$

$$\text{where } f(0^+) = \lim_{t\to 0} f(t), \quad t > 0$$

Review of last lecture

- Complex variables
- Differential equations
- Laplace transform
  - Partial fraction expansion of a rational function
  - Final value theorem
  - Differential theorem
  - Integral theorem

$$L\left[\int_0^t f(\tau)d\tau\right] = \frac{F(s)}{s}$$

  - Shifting theorems

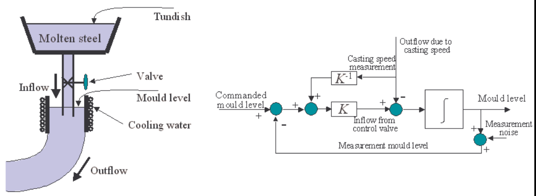
$$L[f(t-t_0)u(t-t_0)] = e^{-t_0 s}F(s)$$
$$L[e^{-at}f(t)] = F(s+a)$$

Review of last lecture

- Complex variables
- Differential equations
- Laplace transform
  - Partial fraction expansion of a rational function
  - Final value theorem
  - Differential theorem
  - Integral theorem
  - Shifting theorems
  - Theorem of convolution integral

$$L^{-1}[F_1(s)F_2(s)] = \int_0^t f_1(t-\tau)f_2(\tau)d\tau = \int_0^t f_1(\tau)f_2(t-\tau)d\tau$$

About Lab 1



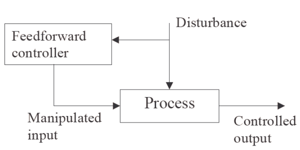
- Feedback and feedforward

About Lab 1

- Feedback and feedforward
- Why use feedforward controller?
  - Feedback controller does not react to a disturbance before a control error has already occurred
  - A simple “drop-and-hold” experiment

About Lab 1

- Feedback and feedforward
- Why use feedforward controller?
  - Feedback controller does not react to a disturbance before a control error has already occurred
  - A simple “drop-and-hold” experiment
  - Feedforward control configuration measures the disturbance directly and takes control action to eliminate its impact on the process output



About Lab 1

- About feedback and feedforward
- Why use feedforward controller?
- Disadvantages of using feedforward controller
  - It requires the identification of all possible disturbances and their direct measurement
  - Not all changes in the parameters of a process can be compensated by a feedforward controller
  - Feedforward control requires a very good model of the process
- Combined feedforward-feedback control

Outline of this lecture

- System modeling
- Block diagrams and signal flow graphs

13

System modeling

- Why we need system modeling?
  – The first step in feedback control system design
  – We need models to predict the effects of control actions (in dynamical system the effects of actions do not occur immediately)

14

System modeling

- Why we need system modeling?
- Definition of mathematical model
  – Mathematical relationships that relate the output of a system to its input
  – It should be understood that no mathematical model of a physical system is exact
  – We generally strive to develop a model that is adequate for the problem at hand without making the model overly complex
    - Examples in statistical physics and biology

15

System modeling

- Definition of mathematical model
  – Mathematical relationships that relate the output of a system to its input
  – It should be understood that no mathematical model of a physical system is exact
  – We generally strive to develop a model that is adequate for the problem at hand without making the model overly complex
  – Example: Hodgkin-Huxley model

$$I = m^3 h G_{\mathrm{Na}}(E - E_{\mathrm{Na}}) + n^4 G_{\mathrm{K}}(E - E_{\mathrm{K}}) + G_{\mathrm{L}}(E - E_{\mathrm{L}})$$

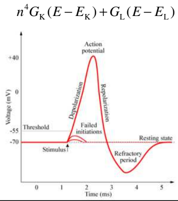
16

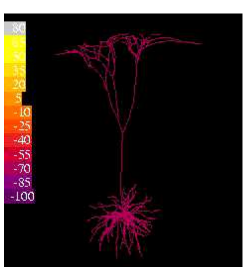
System modeling

- Definition of mathematical model
  – Mathematical relationships that relate the output of a system to its input
  – It should be understood that no mathematical model of a physical system is exact
  – We generally strive to develop a model that is adequate for the problem at hand without making the model overly complex
  – Example:

Hodgkin-Huxley model

$$I = m^3 h G_{\mathrm{Na}}(E - E_{\mathrm{Na}}) +$$
$$n^4 G_{\mathrm{K}}(E - E_{\mathrm{K}}) + G_{\mathrm{L}}(E - E_{\mathrm{L}})$$

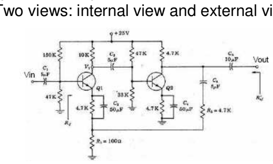




17

System modeling

- Definition of mathematical model
  – Mathematical relationships that relate the output of a system to its input
  – It should be understood that no mathematical model of a physical system is exact
  – We generally strive to develop a model that is adequate for the problem at hand without making the model overly complex
  – Example
  – Models in control engineering ≈ differential equations of dynamical systems
    - Two views: internal view and external view

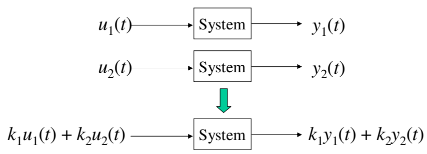


$u(t)$ → System → $y(t)$

18

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
  - A system is linear if superposition applies



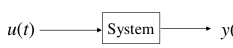
Question: Is $y(t) = u(t) + 2$ a linear system?

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
  - A system is linear if superposition applies
  - A system is time-invariant if its parameters are stationary with respect to time during the system operation
    - An example of time-variant system

System modeling

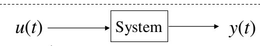
- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions



$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_m\frac{d^m u(t)}{dt^m} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t)$$

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions



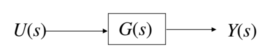
$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_1\frac{dy(t)}{dt} + a_0 y(t) = b_m\frac{d^m u(t)}{dt^m} + \cdots + b_1\frac{du(t)}{dt} + b_0 u(t)$$

Laplace transform (with zero initial conditions)

$$\left(s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0\right)Y(s) = \left(b_m s^m + \cdots + b_1 s + b_0\right)U(s)$$

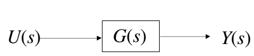
$$\frac{Y(s)}{U(s)} \equiv G(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$

$G(s)$ is called a transfer function



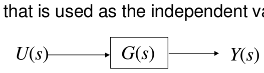
System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
  - The transfer function is defined only for a linear time-invariant system (not for nonlinear systems)
  - The transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input (alternatively, the transfer function between an input variable and an output variable of a system is defined as the Laplace transform of the impulse response)



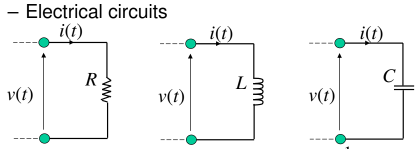
System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
  - The transfer function is defined only for a linear time-invariant system
  - The transfer function between a pair of input and output variables is the ratio of the Laplace transform of the output to the Laplace transform of the input
  - All initial conditions of the system are set to zero
  - The transfer function is independent of the input of the system
  - The transfer function of a continuous-data system is expressed only as a function of the complex variable s. It is not a function of the real variable, time, or any other variable that is used as the independent variable



System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
- Examples
  - Electrical circuits

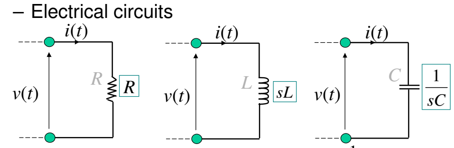


$$v(t)=i(t)R \qquad v(t)=L\frac{di(t)}{dt} \qquad v(t)=\frac{1}{C}\int_0^t i(\tau)d\tau$$

25

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
- Examples
  - Electrical circuits



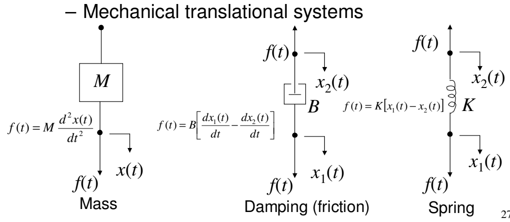
$$v(t)=i(t)R \qquad v(t)=L\frac{di(t)}{dt} \qquad v(t)=\frac{1}{C}\int_0^t i(\tau)d\tau$$

$$\frac{V(s)}{I(s)}=R \qquad \frac{V(s)}{I(s)}=sL \qquad \frac{V(s)}{I(s)}=\frac{1}{sC}$$

26

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
- Examples
  - Electrical circuits
  - Mechanical translational systems

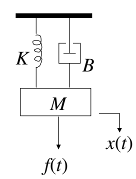


$$f(t)=M\frac{d^2x(t)}{dt^2} \qquad f(t)=B\left[\frac{dx_1(t)}{dt}-\frac{dx_2(t)}{dt}\right] \qquad f(t)=K[x_1(t)-x_2(t)]$$

27

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
- Examples
  - Electrical circuits
  - Mechanical translational systems



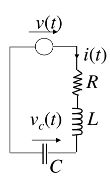
$$M\frac{d^2x}{dt^2}=f(t)-B\frac{dx}{dt}-Kx$$

$$G(s)=\frac{X(s)}{F(s)}=\frac{1}{Ms^2+Bs+K}$$

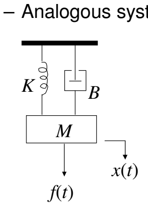
28

System modeling

- Definition of mathematical model
- Definition of linear and time-invariant system
- Transfer functions
- Examples
  - Electrical circuits
  - Mechanical translational systems
  - Analogous systems

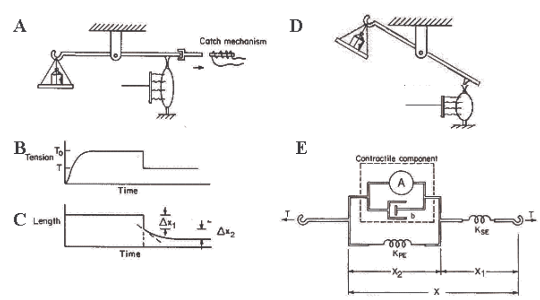


$$G(s)=\frac{V_C(s)}{V(s)}=\frac{1}{LCs^2+RCs+1}$$



$$G(s)=\frac{X(s)}{F(s)}=\frac{1}{Ms^2+Bs+K}$$

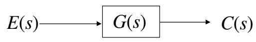
29



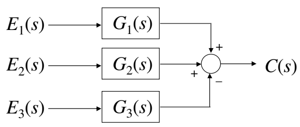
Development of a mathematical muscle model. In this experiment, a muscle is being stimulated when the catch is released. Because there is less weight in the basket than the force that is being produced by the muscle, the muscle shortens rapidly by an amount and then gradually by a different amount. From the changes in the length and tension in the muscle, a muscle model is produced. [From T.A. McMahon (1984)].

Block diagrams

- The transfer function relationship $C(s) = G(s)\, E(s)$ can be graphically denoted through a block diagram



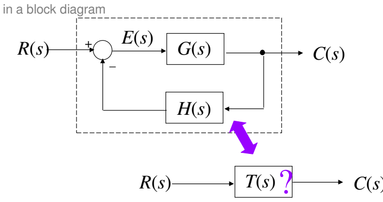
- Summing junction in a block diagram



$C(s) = G_1(s)\, E_1(s) + G_2(s)\, E_2(s) - G_3(s)\, E_3(s)$

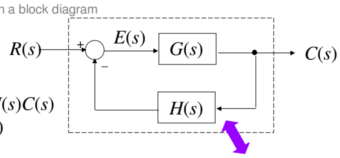
Block diagrams

- The transfer function relationship can be graphically denoted through a block diagram
- Summing junction in a block diagram
- Example:



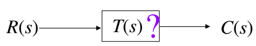
Block diagrams

- The transfer function relationship can be graphically denoted through a block diagram
- Summing junction in a block diagram
- Example:



$$\begin{cases} E(s) = R(s) - H(s)C(s) \\ C(s) = G(s)E(s) \end{cases}$$

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$



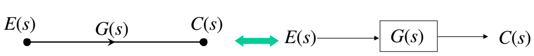
$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Block diagrams

- The transfer function relationship can be graphically denoted through a block diagram
- Summing junction in a block diagram
- Example
- Finding system transfer functions involves solving simultaneously algebra equations
  - by eliminating variables
  - by Cramer's rules
  - by inverse matrix procedures

Signal flow graphs

- A signal flow graph is also used to denote graphically the transfer function relationship
  - Each signal is represented by a node
  - Each transfer function is represented by a branch



Signal flow graphs

- A signal flow graph is also used to denote graphically the transfer function relationship
- A signal flow graph and a block diagram contain exactly the same information (there is no advantage to one over the other; there is only personal preference)

References

- K. J. Astrom and R. M. Murray. Feedback Systems: An Introduction for Scientists and Engineers. Manuscript, 2006.
- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- R. Shadmehr and S. P. Wise. The Computational Neurobiology of Reaching and Pointing: A Foundation for Motor Learning. MIT Press, 2005.
- http://en.wikipedia.org/wiki/Hodgkin-Huxley_model
- http://www.bgu.ac.il/chem_eng/pages/Courses/oren%20courses/Chapter_9.pdf

37