ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 24: Course Review

April 13, 2009

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Announcement

- Times of final exam
  - Wednesday April 15, 9:00 am-11:00 am, or
  - Saturday April 25, 10:00 am-12:00 pm
- Cheat sheet
  - You are allowed to bring to the exam one page (front and back) together with the two pages that you used for Quiz I and Quiz II
- Extra office hours
  - Monday April 13, 11:00 am-1:00 am
  - Tuesday April 14, 3:00 pm-7:00 pm

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Course description (from Lecture 1)

- This course is concerned with both analysis and design of feedback linear control systems
  - Analysis: system modeling, sensitivity, and stability
  - Design: time domain techniques (root locus analysis) and frequency domain techniques (Bode plots and Nyquist theory)
- Introduction to other topics in control systems
  - State-space approaches
  - Nonlinear control

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Outline of this review

- Six concepts
- Five techniques
- Five controllers

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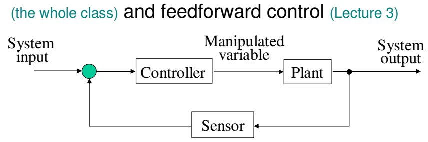
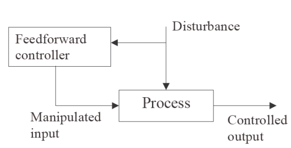
Concepts (1)

- Feedback
  - Open-loop and closed-loop (Lecture 1); feedback control (the whole class) and feedforward control (Lecture 3)





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Concepts (1)

- Feedback
  - Open-loop and closed-loop (Lecture 1); feedback control (the whole class) and feedforward control (Lecture 3)
  - Advantages and disadvantages (Lectures 1 and 6)

> Compared to open-loop control, feedback can be used to reduce steady-state error, reduce the system errors to disturbances, reduce the system's sensitivity to parameter variations, speed up the transient response, and stabilize the system. However, feedback may create instability.

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Concepts (2)

- Transfer function
  - Mathematical modeling (Lectures 2 and 3)
  - LTI system (Lecture 3)
  - Block diagrams (Lecture 3)
  - Open-loop function (Lecture 9) and open-loop transfer function (Lecture 20)

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Concepts (3)

- Time-domain analysis
  - First-order and second order systems (Lectures 4 and 5)
    - Why do we emphasize first-order and second-order systems?
    - Time constant, dc gain, damping ratio, and natural frequency
  - Time responses (Lectures 4 and 5)
    - Step response
    - Ramp response
    - Time response specifications in design (rise time, overshoot, settling time, and steady state value)
  - Steady state accuracy and system type (Lecture 7)
  - Transient response and system modes (Lecture 7)
  - Effect of poles and zeros (Lectures 4 and 5)

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Concepts (4)

- Frequency-domain analysis
  - Why do we study frequency-domain analysis and design?
  - Frequency response function (magnitude change, phase shift, and bandwidth) (Lectures 4, 5, and 13)
  - Visualizing frequency response function (Lecture 13)
  - Design specifications in terms of frequency responses (steady-state accuracy, rise time, settling time, overshoot, etc.) (Lecture 19)

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Concepts (5)

- Stability
  - Stability and maneuverability (Lecture 1)
  - BIBO stability for LTI systems and stability criterion (Lecture 5)
  - Routh stability criterion (not in Final)
  - Relative stability: gain margin and phase margin (Lectures 17 and 18)
  - Relative stability and transient response may exert conflicting requirements in control design (for example, increasing loop gain tends to increase bandwidth, reduce reaction time, but degrade stability margin, Lecture 19)

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Concepts (6)

- Sensitivity (Lecture 6)
  - Definition
  - Disturbance rejection
  - Sensitivity and stability margin may exert conflicting requirements in control design

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Techniques (1)

- Laplace transform and inverse transform (Lecture 2)
  - Partial fraction expansion of a rational function
  - Theorems
    - Final value theorem
    - Differential theorem
    - The Final won't cover the other theorems

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Techniques (2)

- Root locus plot (Lectures 8, 9, and 10)
  - Angle criterion
  - Magnitude criterion
  - Six rules (see next slide)

Six Rules for root locus plot

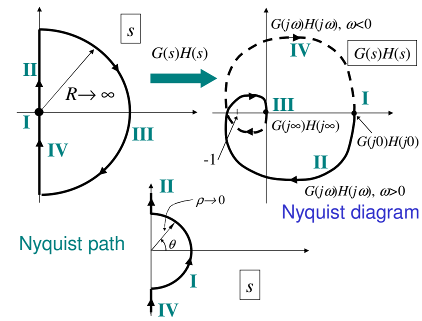
- The root locus is symmetrical with respect to the real axis
- The root locus originates on the poles of $G(s)H(s)$ (for $K = 0$) and terminates on the zeros of $G(s)H(s)$ (as $K \to \infty$), including those zeros at infinity
- If the open-loop function has zeros at infinity, the root locus approaches asymptotes as K approaches infinity. The asymptotes are located at the angles $\theta = r180^\circ / \alpha, \quad \alpha = n - m, \quad r = \pm 1, \pm 3, \ldots$
  and these asymptotes intersect the real axis at the point
  $$\sigma_a = \frac{\text{(sum of finite poles) - (sum of finite zeros)}}{\text{(number of finite poles) - (number of finite zeros)}}$$
- The root locus includes all points on the real axis to the left of an odd number of real critical frequencies (poles and zeros)
- The breakaway points on a root locus will appear among the roots of the polynomial obtained from $N(s)D'(s) - N'(s)D(s) = 0,$
  where $N(s)$ and $D(s)$ are the numerator and denominator polynomials, respectively, of $G(s)H(s)$
- Loci will depart from a pole p_j (arrive at a zero z_j) of $G(s)H(s)$ at the angle θ_d (θ_a), where $\theta_d = \sum_i \theta_{zi} - \sum_{i, i \neq j} \theta_{pi} + r(180^\circ), \; \theta_a = \sum_i \theta_{pi} - \sum_{i, i \neq j} \theta_{zi} + r(180^\circ)$
  and where $r = \pm 1, \pm 3, \ldots$ and θ_{pi} (θ_{zi}) represent the angles from pole p_i (zero z_i), respectively, to p_j (zero z_j)

Techniques (3)

- Bode plot (Lectures 13 and 14)
  - Step 1: Rewrite the transfer function in proper form so that the lowest order term (nonzero) in the numerator and denominator are both unity
  - Step 2: Separate the transfer function into its constituent parts—a constant, poles and zeros at the origin, nonzero real poles and zeros, and complex conjugate poles and zeros
  - Step 3: Draw the Bode diagram for each part
  - Step 4: Draw the overall Bode diagram by adding up the results from Step 3

Techniques (4)

- Nyquist diagram (Lectures 15-18)
  - Nyquist path and Nyquist diagram



Techniques (4)

- Nyquist diagram (Lectures 15-18)
  - Nyquist path and Nyquist diagram
  - Nyquist criterion

The Nyquist path is shown in the left figure. This path is mapped through the open-loop function $G(s)H(s)$ into the Nyquist diagram, as illustrated in the right figure. Then

$$N = Z - P$$

where Z is the number of roots of the system characteristic equation in the right half-plane, N is the number of clockwise encirclements of the -1 point, and P is the number of poles of the open-loop function $G(s)H(s)$ in the right half-plane.

Techniques (5)

- A procedure to derive characteristic equation of the form $1 + G_c(s)G_e(s) = 0$ (Lecture 20)
  - The system input is ignored, and the system is opened at the input of the compensator
  - Find $G_{ol}(s)$ such that

$$E_o(s) = G_{ol}(s)E_i(s)$$

  - The system characteristic equation is then

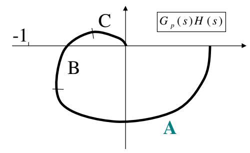
$$1 - G_{ol}(s) = 0$$

Controllers (1)

- Gain compensator (proportional controller) (Lecture 20)
  - Design based on root-locus plot
  - Design based on frequency-domain analysis
    - Effect of gain compensation on Nyquist diagram
    - Effect of gain compensation on Bode plot

Controllers (2)

- Phase-lag compensator
  - Design based on root-locus plot (Lecture 12) (not in Final)
  - Design based on frequency-domain analysis (Lectures 20 and 21)

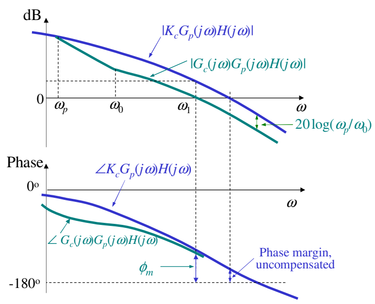


  - Design procedure (see next slide) (Lecture 21)

Phase-lag design procedure

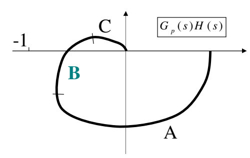
$$G_c(s) = \frac{K_c(1+s/\omega_0)}{1+s/\omega_p}$$

- Adjust the dc gain of $G_p(s)H(s)$ by the factor K_c to satisfy low-frequency specifications
- Find the frequency ω_1 at which the angle of $K_cG_p(s)H(s)$ is equal to $-180^o+\phi_m+5^o$, where ϕ_m is the specified phase margin
- The magnitude of zero is given by $\omega_0 = 0.1\,\omega_1$
- The magnitude of pole is given by $\omega_p = 0.1\,\omega_1 / |K_cG_p(j\omega_1)H(j\omega_1)|$



Controllers (3)

- Phase-lead compensator
  - Design based on root-locus plot (Lecture 12) (not in Final)
  - Design based on frequency-domain analysis (Lecture 22)

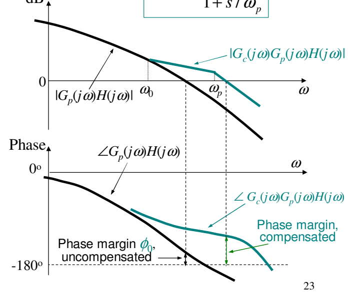


  - Design procedure (see next slide) (Lecture 22)

Phase-lead design procedure

$$G_c(s) = \frac{1+s/\omega_0}{1+s/\omega_p}$$

- Choose a zero location in the vicinity of the 0-dB crossover of $G_p(s)H(s)$
- Choose a ratio of ω_p/ω_0 that gives a value of θ_m larger than ϕ_m-ϕ_0, where ϕ_m is the required phase margin. Calculate ω_p
- Next calculate the compensated Bode diagram, and determine if the phase margin is adequate. If not move the pole in the direction that will adjust the phase margin to the desired value. If moving the pole does not give the desired results, try moving the zero



Controllers (4)

- Lag-lead compensator (Lecture 23)

$$G_c(s) = G_{cg}(s)G_{cd}(s) = K_c\frac{1+s/\omega_{0g}}{1+s/\omega_{pg}}\frac{1+s/\omega_{0d}}{1+s/\omega_{pd}}$$

  - Lag-lead design procedure
    - The phase-lag section of the lag-lead compensator can be designed to maintain the low-frequency gain while realizing a part of the gain margin
    - The phase-lead section of the compensator then realizes the remainder of the phase-margin, while increasing the system band-width to achieve the faster system response

Controllers (5)

- PID controller (Lectures 12, 23, and Supplemental Note)

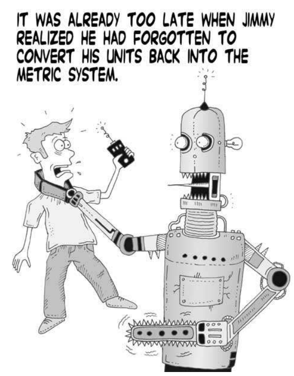
$G_c(s) = K_P + \frac{K_I}{s} + K_D s$ $\quad$ $G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{1 + s/\omega_{pd}}$

  – PID controller can be tuned using the following methods (not in Final)
    - Ziegler-Nichols oscillation method
    - Ziegler-Nichols reaction curve method
    - Cohen-Coon reaction curve method

  – PID controller can be designed by the precedure given for the design of lag-lead compensators:
    - PI section is designed first, in order to realize a part of the specified gain margin; then we get K_P and K_I
    - The remaining parameter, K_D, is then calcuated to realize the total phase margin

Four-wheel drive

It is not always good to have more control...



As a control engineer, “with great power comes great responsibility” [quoted from “Spiderman”].