ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

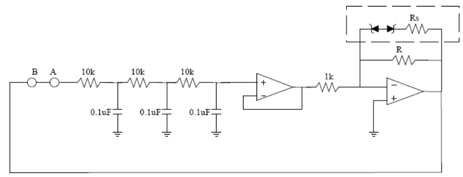
Lecture 23: Frequency Response Design (V)

April 8, 2009

Instructor: Zhi-Hong Mao
Assistant Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

About Lab 4

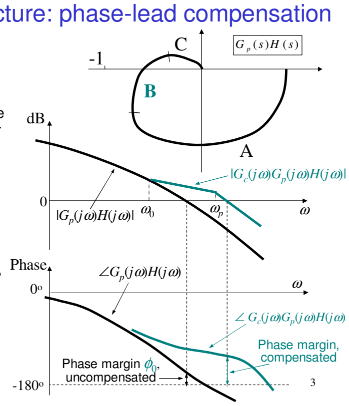
- Lab 4
  - Question: How to model the nonlinear component? in Simulink?



Review of last lecture: phase-lead compensation

$$G_c(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}$$

- Choose a zero location in the vicinity of the 0-dB crossover of $G_p(s)H(s)$
- Choose a ratio of ω_p/ω_0 that gives a value of θ_m larger than $\phi_m - \phi_0$, where ϕ_m is the required phase margin. Calculate ω_p
- Next calculate the compensated Bode diagram, and determine if the phase margin is adequate. If not move the pole in the direction that will adjust the phase margin to the desired value. If moving the pole does not give the desired results, try moving the zero



Outline of this lecture

- Lag-lead compensation
- PID controller design
  - PI controller
  - PD controller
  - PID controller
- Concepts in modern control: state-variable modeling, observability, and controllability

Lag-lead compensation

- Why use lag-lead compensation?
  - Using phase-lead compensation may result in large high-frequency gain (to achieve a desirable phase margin, the compensator pole-to-zero ratio may have to be large); this can (1) induce high-frequency noise problems and (2) generate large signals
  - Using phase-lag compensation may lead to a slow system response
  - Lag-lead controller offers much more flexibility than does either the phase-lag or phase-lead controller separately

Lag-lead compensation

- Why use lag-lead compensation?
- Lag-lead compensator

$$G_c(s) = G_{cg}(s)G_{cd}(s) = K_c \frac{1 + s/\omega_{0g}}{1 + s/\omega_{pg}} \frac{1 + s/\omega_{0d}}{1 + s/\omega_{pd}}$$

- Lag-lead design procedure
  - The phase-lag section of the lag-lead compensator can be designed to maintain the low-frequency gain while realizing a part of the phase margin
  - The phase-lead section of the compensator then realizes the remainder of the phase margin, while increasing the system band-width to achieve the faster system response

PID controller design

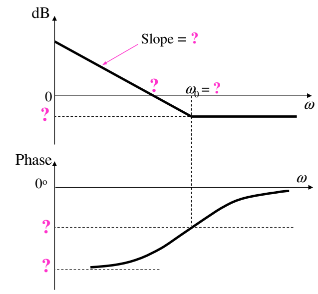
- Most commercially available controllers are PID controllers
- PID controller can be considered as a form of lag-lead controller

$$G_c(s) = K_P + \frac{K_I}{s} + K_D s \qquad G_c(s) = K_P + \frac{K_I}{s} + \frac{K_D s}{1 + s/\omega_{pd}}$$

  - Proportional controller
  - PI controller
  - PD controller
  - PID controller

PI controller

$$G_c(s) = K_P + \frac{K_I}{s} = \frac{K_P s + K_I}{s}$$



PI controller

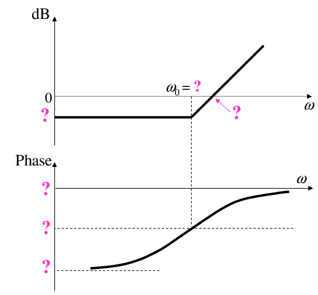
- PI controller design procedure
  - Adjust the dc gain of $G_p(s)H(s)$ by the factor K_c to satisfy low-frequency specifications
  - Find the frequency ω_1 at which the angle of $K_c G_p(s)H(s)$ is equal to $-180^o + \phi_m + 5^o$
  - The gain K_P is then given by

    $$K_P = 1 / |K_c G_p(j\omega_1) H(j\omega_1)|$$
  - The magnitude of zero is given by $\omega_0 = K_I/K_P = 0.1\omega_1$ and thus $K_I = 0.1\omega_1 K_P$
  - The controller transfer function is then

    $$G_c(s) = K_c(K_P + K_I/s)$$

PD controller

$$G_c(s) = K_P + K_D s = K_P(1 + s/\omega_0)$$



PD controller

- PD controller design procedure
  - Choose a zero location in the vicinity of the 0-dB crossover of $G_p(s)H(s)$
  - Calculate the compensated Bode diagram, and determine if the phase margin is adequate. If not move the zero in the direction that will adjust the phase margin to the desired value

PID controller

- PID controller design procedure
  - The PID controller can be designed by the procedure given for the design of lag-lead compensators:
    - PI section is designed first, in order to realize a part of the specified gain margin; then we get K_P and K_I
    - The remaining parameter, K_D, is then calculated to realize the total phase margin

State-variable modeling

- Definition of state
  - The state of a system at any time t_0 is the amount of information at t_0 that, together with all inputs for $t \geq t_0$, uniquely determines the behavior of the system for all $t \geq t_0$

13

State-variable modeling

- Definition of state
- The standard form of the state equations of a LTI analog system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$
where
$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}, \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \mathbf{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix}, \mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$$

14

State-variable modeling

- Definition of state
- The standard form of the state equations of a LTI analog system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

- Observability
  - Observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs
  - A system with n states is observable if the rank of observability matrix $\begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$ is equal to n

15

State-variable modeling

- Definition of state
- The standard form of the state equations of a LTI analog system

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

- Observability
- Controllability
  - Controllability denotes the ability to move a system around in its entire configuration space using only certain admissible manipulations
  - A system with n states is controllable if the rank of the matrix $\left[A\; AB\; AB^2 \cdots AB^{n-1}\right]$ is equal to n

16

State-variable modeling

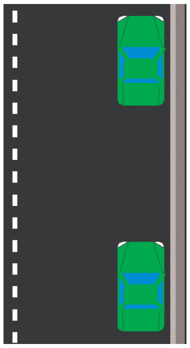
- Definition of state
- The standard form of the state equations of a LTI analog system
- Observability
- Controllability
- Examples about controllability
  - Control of the movement of pinna (auricle)

17

State-variable modeling

- Definition of state
- The standard form of the state equations of a LTI analog system
- Observability
- Controllability
- Examples about controllability
  - Control of the movement of pinna (auricle)
  - Can you park your car at any location?

18



19

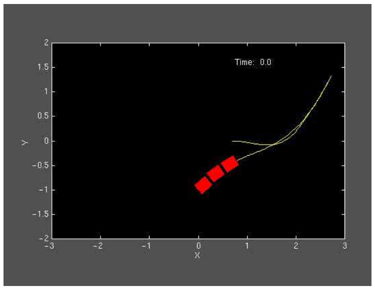
20

21

22

Another way of parallel parking

23



Parking of a trailer

24

References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- http://www.answers.com/topic/parallel-parking
- http://www.carbuyingtips.com/pics/crash66.jpg
- http://www.virtualmex.com/album/Lake%20Chapala/Ajijic_Parallel_Parking.JPG

25