ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 22: Frequency Response Design (IV)

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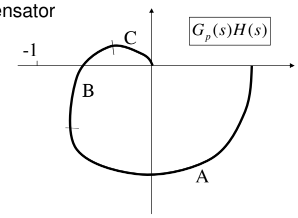
Review of last lecture

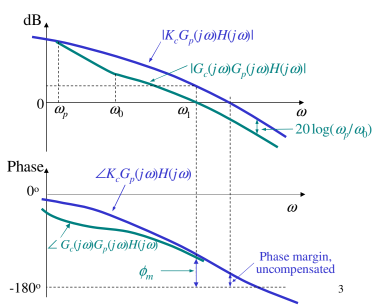
- Basic idea of phase-lag design
  - Placement of the pole and zero of the phase-lead compensator



Question: In which region (A, B, or C) should we put the zero and pole of the phase-lag compensator?

Review of last lecture

- Basic idea of phase-lag design
- Phase-lag design procedure
  - Adjust the dc gain of $G_p(s)H(s)$ by the factor K_c to satisfy low-frequency specifications
  - Find the frequency ω_1 at which the angle of $K_cG_p(s)H(s)$ is equal to $-180^o+\phi_m+5^o$, where ϕ_m is the specified phase margin
  - The magnitude of zero is given by $\omega_0 = 0.1\,\omega_1$
  - The magnitude of pole is given by $\omega_p = 0.1\,\omega_1 / |K_cG_p(j\omega_1)H(j\omega_1)|$



Review of last lecture

- Basic idea of phase-lag design
- Phase-lag design procedure
- Advantages and disadvantages of phase-lag compensation
  - Advantages
    - The low-frequency characteristics are improved
    - Stability margins are maintained or improved
    - Bandwidth is reduced, which is an advantage if high-frequency noise is a problem
  - Disadvantages
    - Reduced bandwidth is a disadvantage in some systems
    - The system transient response will have one very slow term

Outline of this lecture

- Phase-lead basics
- Basic idea of phase-lead design
- Phase-lead design procedure
- Advantages and disadvantages of phase-lead compensation
- Concepts in modern control: nonlinear control

Phase-lead basics

- Phase-lead compensator has the transfer function

$$G_c(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}$$

Question 1: Which one is bigger, ω_0 or ω_p?

Question 2: What is the Bode diagram for this compensator?

Question 3: Is it a low-pass filter or high-pass filter?

Question 4: What is the gain augmentation at high frequency?

Question 5: What is the frequency at which the maximum phase lead occur?

Phase-lead basics

- Phase-lead compensator has the transfer function

$$G_c(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}$$

Question 5: What is the frequency at which the maximum phase lead occur?

$$\omega_m = \sqrt{\omega_p \omega_0}$$

Question 6: What is the maximum phase-lead?

$$\theta_m = \tan^{-1}\sqrt{\frac{\omega_p}{\omega_0}} - \tan^{-1}\sqrt{\frac{\omega_0}{\omega_p}}$$

$$\tan\theta_m = \frac{1}{2}\left(\sqrt{\frac{\omega_p}{\omega_0}} - \sqrt{\frac{\omega_0}{\omega_p}}\right)$$

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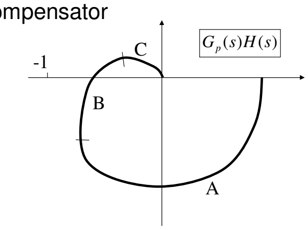
Basic idea of phase-lead design

- Placement of the pole and zero of the phase-lead compensator



Question: In which region (A, B, or C) should we put the zero and pole of the phase-lead compensator?

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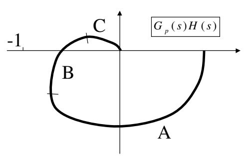
Basic idea of phase-lead design

- Placement of the pole and zero of the phase-lead compensator
  - In region A: decreased stability margins
  - In region C: little effect
  - In region B: increased stability margins due to added phase-lead effect



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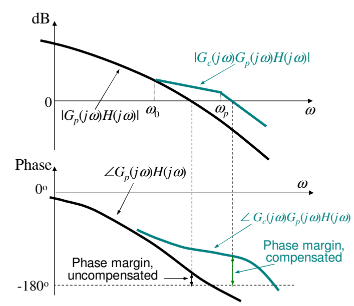
Phase-lead design procedure

$$G_c(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}$$

- Choose a zero location in the vicinity of the 0-dB crossover of $G_p(s)H(s)$



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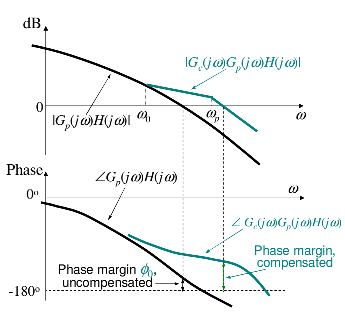
Phase-lead design procedure

$$G_c(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}$$

- Choose a zero location in the vicinity of the 0-dB crossover of $G_p(s)H(s)$
- Choose a ratio of ω_p/ω_0 that gives a value of θ_m larger than $\phi_m - \phi_0$, where ϕ_m is the required phase margin
- Calculate ω_p



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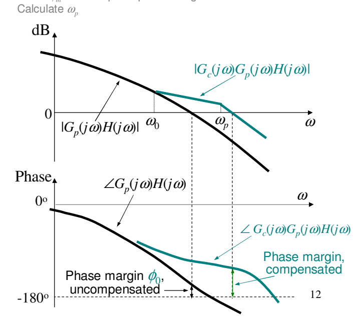
Phase-lead design procedure

$$G_c(s) = \frac{1 + s/\omega_0}{1 + s/\omega_p}$$

- Choose a zero location in the vicinity of the 0-dB crossover of $G_p(s)H(s)$
- Choose a ratio of ω_p/ω_0 that gives a value of θ_m larger than ϕ_m-ϕ_0, where ϕ_m is the required phase margin
- Calculate ω_p

- Next calculate the compensated Bode diagram, and determine if the phase margin is adequate. If not move the pole in the direction that will adjust the phase margin to the desired value. If moving the pole does not give the desired results, try moving the zero



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Advantages and disadvantages of phase-lead compensation

- Advantages
  - Improved stability margin
  - Improved high-frequency performance (such as speed of response)
- Disadvantages
  - Accentuated high-frequency noise problems
  - May generate large signals, which may damage the system or at least result in nonlinear operation of the system

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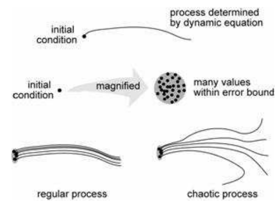
Nonlinear control

- Nonlinear phenomena
  - Finite escape time
  - Multiple isolated equilibria (associate memory in neural networks)
  - Limit cycles (recall Lab 4)
  - Chaos



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Nonlinear control

- Nonlinear phenomena
- Important concepts in control of nonlinear systems
  - Design based on linearized model
  - Switched linear controllers (recall the hybrid control)
  - Nonlinear controller (e.g. neural networks)
  - Lyapunov stability

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An example of chaotic behavior in a pendulum system

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Can an inverted pendulum be stabilized by using vertical force only? [Demo]

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References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- http://monet.unibas.ch/~elmer/pendulum/vpend.htm
- http://www.fas.harvard.edu/~scdiroff/lds/MathamaticalTopics/ChaoticPendulum/ChaoticPendulum.html

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