

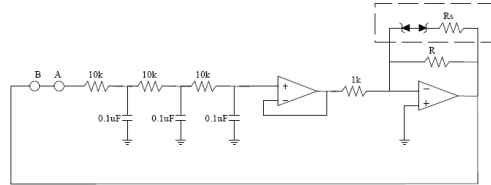
Lecture 21: Frequency Response Design (III)

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1

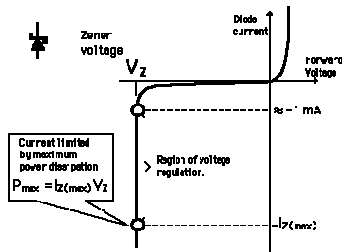
About Lab 4



2

About Lab 4

- Zener diode



3

About Lab 4

- Zener diode
- How to get oscillation?
 - Conditions for a **positive** feedback system with no phase lag to be stable
 - Conditions for a **negative** feedback system with a phase lag of 180 degrees to be stable (a revisit of Nyquist criterion)

4

About Lab 4

- Zener diode
- How to get oscillation?
 - Conditions for a positive feedback system with no phase lag to be stable
 - Conditions for a negative feedback system with a phase lag of 180 degrees to be stable (a revisit of Nyquist criterion)
 - **Examples:** (1) Strategies for pumping a swing; (2) a spider's oscillatory responses to web motion

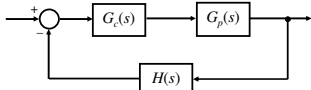
5



6

Review of last lecture

- Cascade compensation
 - How to derive system characteristic equation of the form $1 + G_c(s)G_p(s) = 0$
 - Open-loop transfer function and open-loop function



Question: What is the open-loop transfer function of the above system?

7

Review of last lecture

- Cascade compensation
- Gain compensation

Exercise: For the following system, determine the maximum value for the gain K_B which will result in a gain margin of 6 dB or more and a phase margin of 45 degrees or more:

$$G_c(s)G_p(s)H(s) = \frac{K_B}{s(1+s/5)^2}$$

8

Review of last lecture

- Cascade compensation
- Gain compensation
- Phase-lag compensation

$$G_c(s) = \frac{1 + s/\omega_z}{1 + s/\omega_p}$$

Question 1: What is the gain reduction at high frequency?

Question 2: What is the frequency at which the maximum phase lag occur?

9

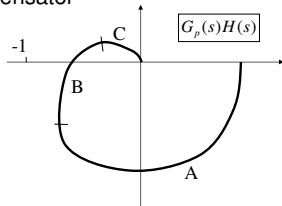
Outline of this lecture

- Basic idea of phase-lag design
- Phase-lag design procedure
- An example
- Advantages and disadvantages of phase-lag compensation
- Concepts in modern control: hybrid control

10

Basic idea of phase-lag design

- Placement of the pole and zero of the phase-lag compensator

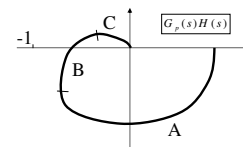


Question: In which region (A, B, or C) should we put the zero and pole of the phase-lag compensator?

11

Basic idea of phase-lag design

- Placement of the pole and zero of the phase-lag compensator
 - In region C: little effect
 - In region B: destabilizing due to the added phase-lag
 - In region A: little effect on stability from the added phase-lag and increased stability margins due to reduced gain in the higher frequency region



12

Phase-lag design procedure

$$G_c(s) = \frac{K_c(1+s/\omega_p)}{1+s/\omega_p}$$

- Adjust the dc gain of $G_p(s)H(s)$ by the factor K_c to satisfy low-frequency specifications

13

Phase-lag design procedure

$$G_c(s) = \frac{K_c(1+s/\omega_0)}{1+s/\omega_p}$$

- Adjust the dc gain of $G_c(s)H(s)$ by the factor K_c to satisfy low-frequency specifications
- Find the frequency ω_1 at which the angle of $K_c G_p(s)H(s)$ is equal to $-180^\circ + \phi_m + 5^\circ$, where ϕ_m is the specified phase margin

14

Phase-lag design procedure

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- Adjust the dc gain of $G_c(s)H(s)$ by the factor K_c to satisfy low-frequency specifications
- Find the frequency ω_1 at which the angle of $K_c G_p(s)H(s)$ is equal to $-180^\circ + \phi_m + 5^\circ$, where ϕ_m is the specified phase margin

The magnitude of zero is given by $\omega_0 = 0.1\omega_1$

The magnitude of pole is given by $\omega_p = 0.1\omega_1 / |K_c G_p(j\omega_1)H(j\omega_1)|$

15

An example

ω	dB	Phase
0.1	26.0	-98.6
0.2	19.8	-107.0
0.3	16.0	-115.2
0.4	13.2	-123.1
0.5	10.8	-130.6
0.6	8.75	-137.7
0.7	6.88	-144.3
0.8	5.17	-150.5
0.9	3.56	-156.2
1.0	2.04	-161.6
1.1432	0.00005	-168.6
1.4142	-3.52	-180.0
2.0	-10.0	-198.4
3.0	-18.6	-217.9
4.0	-25.3	-229.4
5.0	-30.7	-236.9
6.0	-35.2	-242.1
7.0	-39.1	-245.9
8.0	-42.5	-248.8
9.0	-45.5	-251.1
10.0	-48.2	-253.0

Consider a system with the open-loop function $G_p(s)H(s) = \frac{4}{s(s+1)(s+2)}$ and assume that the low-frequency gain meets the system specifications (thus $K_c=1$). Design a phase-lag compensator satisfy the requirement of a phase margin of 50 degrees.

16

An example

ω	dB	Phase
0.1	26.0	-98.6
0.2	19.8	-107.0
0.3	16.0	-115.2
0.4	13.2	-123.1
0.5	10.8	-130.6
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$$G_p(s)H(s) = \frac{4}{s(s+1)(s+2)}$$

Find frequency ω_1 :

$$\angle G_p(j\omega_1)H(j\omega_1) = -180 + 50^\circ + 5^\circ = -125^\circ$$

$$\omega_1 \approx 0.4$$

Calculate ω_0, ω_p :

$$\omega_0 = 0.1\omega_1$$

$$\omega_p = 0.1\omega_1 / |K_c G_p(j\omega_1)H(j\omega_1)|$$

17

An example

$$G_p(s)H(s) = \frac{4}{s(s+1)(s+2)}$$

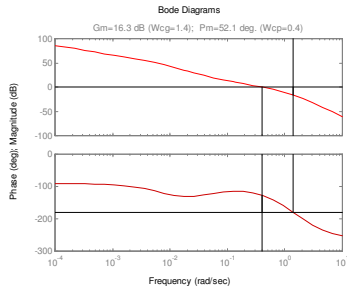
$$G_c(s) = \frac{1+s/0.04}{1+s/0.0088} = \frac{1+25s}{1+1136s}$$

Matlab program:

```
Gp = tf([4], [1 3 2 0]);
Gc = tf([25 1], [113.6 1]);
margin(Gp*Gc)
[Gm, Pm, Wcg, Wcp]=margin(Gp*Gc)
```

18

An example



19

Advantages and disadvantages of phase-lag compensation

- Advantages
 - The low-frequency characteristics are improved
 - Stability margins are maintained or improved
 - Bandwidth is reduced, which is an advantage if high-frequency noise is a problem
- Disadvantages
 - Reduced bandwidth is a disadvantage in some systems
 - The system transient response will have one very slow term

20

Hybrid control

- Hybrid systems combine both “discrete” and “continuous” dynamics: The dynamical evolution is given by equations of motion that depend on both continuous and discrete variables

21

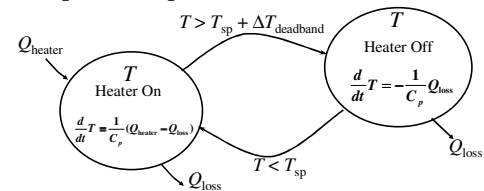
Hybrid control

- Hybrid systems combine both “discrete” and “continuous” dynamics

Examples

- Room with thermostat and heater

$$S = \begin{bmatrix} T \\ \text{Heater Status} \end{bmatrix} \in \mathfrak{R} \times \{\text{On, Off}\} \quad T = \text{Room temperature}$$



22

Hybrid control

- Hybrid systems combine both “discrete” and “continuous” dynamics

Examples

- Room with thermostat and heater
- Still remember the “ankle strategy” and “hip strategy” for balancing (cerebellar control)?
- Exoskeleton and Hybrid Assistive Limb (HAL): HAL has the hybrid control systems which consist of the autonomous controller such as posture control and the comfortable power assist controller based on biological feedback and predictive feedforward

23



24



From Tsukuba University

25



From movie "Alien"

26

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27