ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 18: Recitation

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Exercise 1: Find Bode plot and Nyquist diagram for

$$G(s)H(s)=\frac{-s}{(s-1)(s+1)}$$

Exercise 2: Find Bode plot for

$$G(s)H(s)=\frac{100s}{(s+1)(s+10)}$$

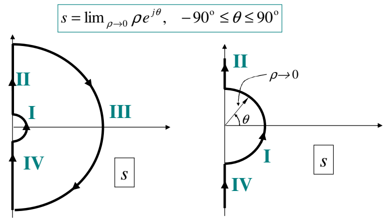
Exercise 3: Find Nyquist diagram for

$$G(s)H(s)=\frac{8s}{(s+1)^2}$$

Review of last lecture

- A special case of Nyquist diagram: poles at the origin
  - Reform the Nyquist path to detour around the origin if the open-loop function has a pole there
  - The detour is chosen to be circular with a radius that approaches zero in the limit

$$s=\lim_{\rho\to 0}\rho e^{j\theta},\quad -90^{\circ}\le\theta\le 90^{\circ}$$



Exercise 4

$$G(s)=\frac{K}{s(s+1)},\ H(s)=1$$

Exercise 5

$$G(s)=\frac{K}{s^2(s+1)},\ H(s)=1$$

Review of last lecture

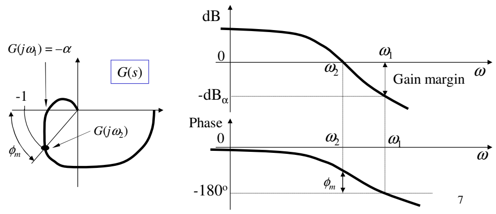
- A special case of Nyquist diagram: poles at the origin
- Some observations
  - Any negative angle added to the Nyquist diagram tends to rotate the diagram towards the -1 point and increases the probability of encirclements of this point
  - Stability can be adversely affected by the addition of negative angles to the diagram, and thus addition of poles at the origin can be destabilizing (why?) (can you give us one more example of the addition of negative angles to the Nyquist diagram?)

Review of last lecture

- A special case of Nyquist diagram: poles at the origin
- Some observations
- A procedure for counting the number of encirclements of the -1 point
  - Draw a ray from the -1 point in any convenient direction
  - The number of clockwise encirclements of the -1 point is equal to the number of crossings of this ray by the Nyquist diagram, in the clockwise direction, minus the number of crossings of the ray in the counterclockwise direction

Review of last lecture

- A special case of Nyquist diagram: poles at the origin
- Some observations
- A procedure for counting the number of encirclements of the -1 point
- Relative stability
  - Two measures of relative stability: gain margin and phase margin (what do they mean?)



7

X-29 at high angle of attack

8

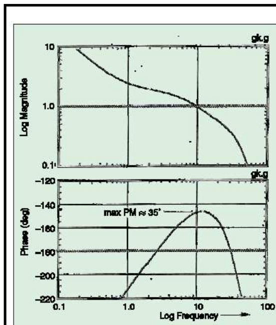
X-29 aircraft stability margins

- Limitations of X-29
  - At one flight condition the model has the following non-minimum phase component

$$G_{\text{nmp}}(s) = \frac{s-26}{s-6}$$

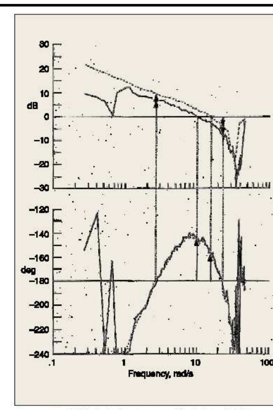
  - The achievable phase margin is less than 45 degrees (note that one of the design criteria was that the phase margin should be greater than 45 degrees for all flight conditions)

9



Prototype Bode diagram for the X-29.

From Gunter Stein's Bode Lecture



X-29 flight data (courtesy Mr. J. Gera, NASA).

Practice problems for Quiz II

11

Problem 1: Plot Nyquist diagrams for

$$G_1(s)H_1(s) = \frac{1}{s(s-1)}$$

$$G_2(s)H_2(s) = \frac{e^{-Ts}}{s+p}, \quad p > 0$$

12

Problem 2: Determine the gain and phase margins for the system with the following open function

$$G_1(s)H_1(s) = \frac{1}{s}$$

$$G_2(s)H_2(s) = Ke^{-Ts}, \quad 0 < K < 1$$

13

Problem 3: Construct the Bode diagrams for the open-loop transfer function

$$G(s)H(s) = \frac{2(s+2)}{s^2 - 1}$$

14

References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- G. Stein. Respect the Unstable. Hendrik W. Bode Lecture at the IEEE Conference on Decision and Control in Tampa, Florida, December 1989 (reprinted in IEEE Control Systems Magazine, 2003).
- http://www.bostondynamics.com/
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15