ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 17: More Examples of Nyquist Diagram; Relative Stability

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Instructor: Zhi-Hong Mao
Assistant Professor of Electrical and Computer Engineering
University of Pittsburgh, Pittsburgh, PA

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About Quiz II, Lab 4, and Homework 6, and Homework 7

- Quiz II
  - Exam date: March 25 (Wednesday)
  - You may take a sheet of note with you

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About Quiz II, Lab 4, Homework 6, and Homework 5

- Quiz II
- Homework 7
  - Problems 8.10 (a) and (b) and 8.13 (a)-(c) in the text book
  - Due March 25 (Wednesday)
- Homework 6
  - Bode diagrams for $G(s)$ and $-G(s)$

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What to be covered in Quiz II

- Phase-lead and phase-lag compensators
- Bode plot
- Nyquist diagram
- Relative stability

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Review of last two lectures

- Cauchy's principle of argument

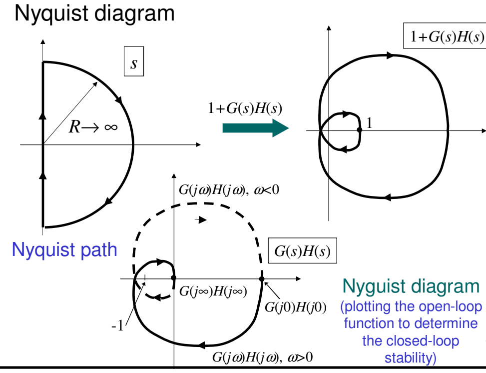
Let $F(s)$ be the ratio of two polynomials in s. Let the closed curve C in the s-plane be mapped into the complex plane through the mapping $F(s)$. If $F(s)$ is analytic (complex differentiable) within and on C, except at a finite number of poles, and if $F(s)$ has neither poles nor zeros on C, then

$$N = Z - P$$

where Z is the number of zeros of $F(s)$ in C, P is the number of poles of $F(s)$ in C, and N is the number of encirclements of the origin, taken in the same sense as C.

Review of last two lectures

- Cauchy's principle of argument
- Nyquist diagram



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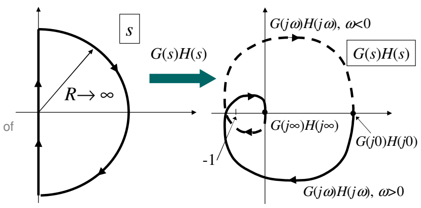
Review of last two lectures

- Cauchy's principle of argument
- Nyquist diagram
- Nyquist criterion



Nyquist path   Nyquist diagram

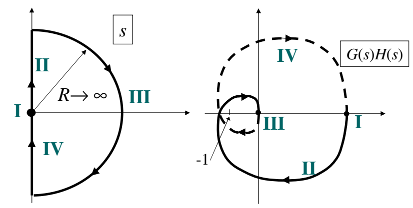
The Nyquist path is shown in the left figure. This path is mapped through the open-loop function $G(s)H(s)$ into the Nyquist diagram, as illustrated in the right figure. Then

$$N = Z - P$$

where Z is the number of roots of the system characteristic equation in the right half-plane, N is the number of clockwise encirclements of the -1 point, and P is the number of poles of the open-loop function $G(s)H(s)$ in the right half-plane.

Review of last two lectures

- Cauchy's principle of argument
- Nyquist diagram
- Nyquist criterion
- Procedure to draw Nyquist diagram
  - The four parts of Nyquist path



Nyquist path   Nyquist diagram

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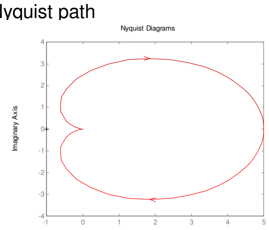
Review of last two lectures

- Cauchy's principle of argument
- Nyquist diagram
- Nyquist criterion
- Procedure to draw Nyquist diagram
  - The four parts of Nyquist path
  - Example:

$$G(s)H(s) = \frac{5}{(s+1)^2}$$



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Outline of this lecture

- A special case of Nyquist diagram: poles at the origin
- Relative stability
- An example about relative stability: X-29 aircraft (with forward swept wings) might not be controlled with sufficient stability margins

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Poles at the origin

- A special case: poles at the origin
  - Consider the case where the open-loop function has a pole or multiple poles at the origin, for example

$$G(s) = \frac{50}{s(s+5)}$$

  - The procedure for analyzing this case also applies to the case of marginally stable open-loop systems with poles on the $j\omega$-axis not equal to zero

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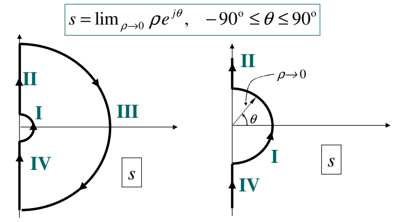
Poles at the origin

- A special case: poles at the origin
- Procedure for analyzing this case
  - Reform the Nyquist path to detour around the origin if the open-loop function has a pole there
  - The detour is chosen to be circular with a radius that approaches zero in the limit

$$s = \lim_{\rho\to 0} \rho e^{j\theta}, \quad -90^\circ \le \theta \le 90^\circ$$



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Poles at the origin

- A special case of poles at the origin
- Procedure for analyzing this case
- Exercise 1

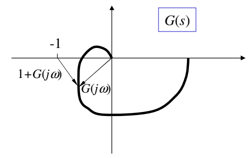
$$G(s)=\frac{K}{s(s+1)},\ H(s)=1$$

- Exercise 2

$$G(s)=\frac{K}{s^2(s+1)},\ H(s)=1$$

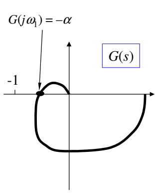
Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the closeness of the Nyquist diagram to the -1 point in the complex plane



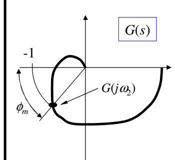
Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the *closeness* of the Nyquist diagram to the -1 point in the complex plane
- Two measures of relative stability
  - Gain margin: If the magnitude of the open-loop function of a stable closed-loop system at -180^o crossover on the Nyquist diagram is the value α, the gain margin is $1/\alpha$
    - The margin is usually given in decibels
    - Phase crossover frequency is defined as the frequency where the phase shift of $G(j\omega)H(j\omega)$ is -180^o (in other words, the gain margin is defined at the phase crossover frequency)



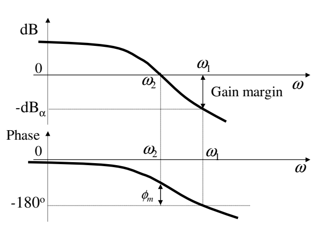
Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the *closeness* of the Nyquist diagram to the -1 point in the complex plane
- Two measures of relative stability
  - Gain margin
  - Phase margin: the magnitude of the minimum angle (ϕ_m) by which the Nyquist diagram must be rotated in order to intersect the -1 point for a stable closed-loop system
    - Gain crossover frequency is defined as the frequency where the magnitude of $G(j\omega)H(j\omega)$ is 0 dB or 1 in absolute value (in other words, the phase margin is defined at the gain crossover frequency)



Relative stability

- Generally we require not only that a system be stable but also that it be stable by some margin of safety
- Relative stability is defined in terms of the *closeness* of the Nyquist diagram to the -1 point in the complex plane
- Two measures of relative stability
- The gain and phase margins may be determined from a Bode diagram



X-29 aircraft stability margins

- X-29 is an experimental aircraft with forward swept wings

X-29 aircraft stability margins

- X-29 is an experimental aircraft with forward swept wings
- Features of X-29
  - Slight instability at supersonic speeds but extreme instability at subsonic speeds
  - Exceptional maneuverability
  - High performance at high angles of attack

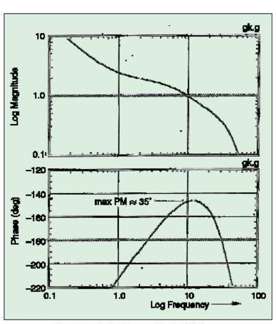
X-29 at high angle of attack

X-29 aircraft stability margins

- X-29 is an experimental aircraft with forward swept wings
- Features of X-29
- Limitations of X-29
  - At one flight condition the model has the following non-minimum phase component

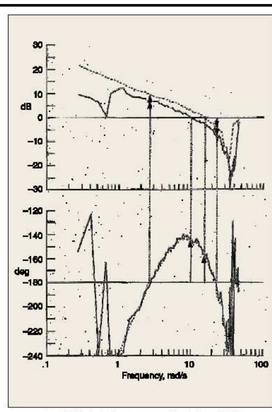
$$G_{\mathrm{nmp}}(s) = \frac{s-26}{s-6}$$

  - The achievable phase margin is less than 45 degrees (note that one of the design criteria was that the phase margin should be greater than 45 degrees for all flight conditions)



Prototype Bode diagram for the X-29.

From Gunter Stein's Bode Lecture



X-29 flight data (courtesy Mr. J. Gera, NASA).

References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- G. Stein. Respect the Unstable. Hendrik W. Bode Lecture at the IEEE Conference on Decision and Control in Tampa, Florida, December 1989 (reprinted in IEEE Control Systems Magazine, 2003).
- http://www.nasa.gov/centers/dryden/news/FactSheets/FS-008-DFRC.html