ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 16: Examples of Nyquist Diagram

March 4, 2009

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Announcement

- No class next week (spring break)

Review of last lecture

- What is Nyquist criterion about?
  - Nyquist criterion allows us to determine the stability of the closed-loop system from a knowledge of the frequency-response of the open-loop function
  - Nyquist diagram can be evaluated either mathematically if the open-loop function is available or experimentally if the physical system is available for experimentation (in the latter case, the open-loop system needs to be stable)
  - Nyquist diagram gives us important information concerning the type of compensation required to stabilize certain types of systems

Review of last lecture

- What is Nyquist criterion about?
- Cauchy's principle of argument

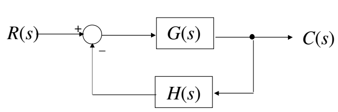
Let $F(s)$ be the ratio of two polynomials in s. Let the closed curve C in the s-plane be mapped into the complex plane through the mapping $F(s)$. If $F(s)$ is analytic (complex differentiable) within and on C, except at a finite number of poles, and if $F(s)$ has neither poles nor zeros on C, then

$$N = Z - P$$

where Z is the number of zeros of $F(s)$ in C, P is the number of poles of $F(s)$ in C, and N is the number of encirclements of the origin, taken in the same sense as C.

Review of last lecture

- What is Nyquist criterion about?
- Cauchy's principle of argument
- Basic idea of Nyquist criterion



Question 1: What are definitions of Nyquist path and Nyquist diagram?

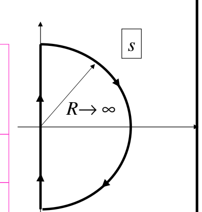
Review of last lecture

- What is Nyquist criterion about?
- Cauchy's principle of argument
- Basic idea of Nyquist criterion

Question 1: In Cauchy's principle of argument, let $F(s)$ be $1 + G(s)H(s)$ and curve C be as shown in the right figure. Then what do Z, P, and N represent?

Question 2: For what value of Z will the closed-loop system be stable?

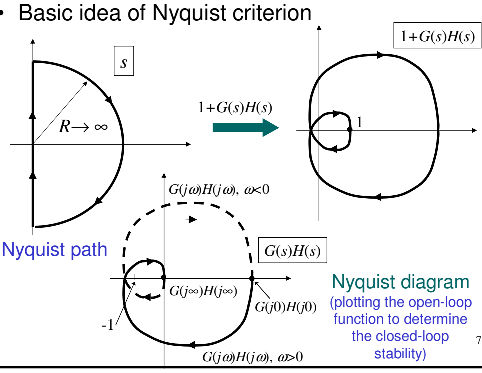
Question 3: For what value of N will the closed-loop system be stable?



Nyquist path

Review of last lecture

- What is Nyquist criterion about?
- Cauchy's principle of argument
- Basic idea of Nyquist criterion



Nyquist path

Nyquist diagram
(plotting the open-loop function to determine the closed-loop stability)

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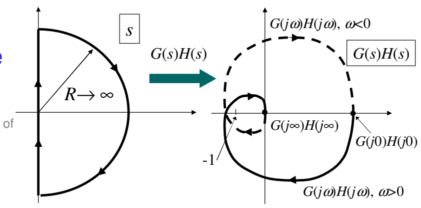
Review of last lecture

- What is Nyquist criterion about?
- Cauchy's principle of argument
- Nyquist diagram
- Nyquist criterion



Nyquist path Nyquist diagram

The Nyquist path is shown in the left figure. This path is mapped through the open-loop function $G(s)H(s)$ into the Nyquist diagram, as illustrated in the right figure. Then

$$N = Z - P$$

where Z is the number of roots of the system characteristic equation in the right half-plane, N is the number of clockwise encirclements of the -1 point, and P is the number of poles of the open-loop function $G(s)H(s)$ in the right half-plane.

Outline of this lecture

- Four parts of Nyquist path
- Examples of Nyquist diagram
- More about Lab 3: dealing with time delay

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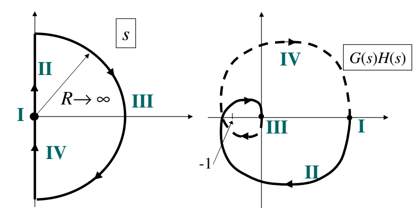
Four parts of Nyquist path

- Part **I**: the origin of the s-plane, which corresponds to $G(0)H(0)$ (or dc gain of the open-loop function) in Nyquist diagram



Nyquist path Nyquist diagram

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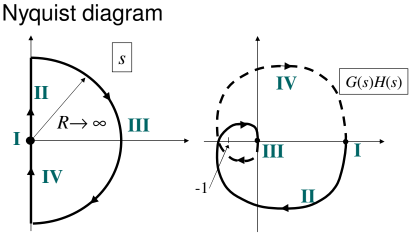
Four parts of Nyquist path

- Part **II**: the positive half of the imaginary axis, which corresponds to $G(j\omega)H(j\omega)$, $\omega > 0$ (frequency response of the open-loop function) in Nyquist diagram



Nyquist path Nyquist diagram

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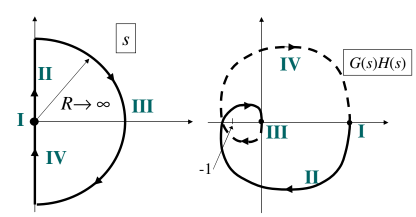
Four parts of Nyquist path

- Part **III**: the infinite arc. Since physical systems are low-pass in nature, the open-loop function evaluated along this arc is zero



Nyquist path Nyquist diagram

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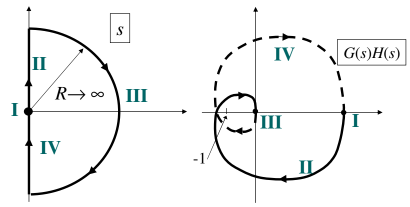
Four parts of Nyquist path

- Part **IV**: the negative half of the imaginary axis, which corresponds to $G(j\omega)H(j\omega)$, $\omega < 0$ (the complex conjugate of the open-loop function evaluated along Part **II**) in Nyquist diagram

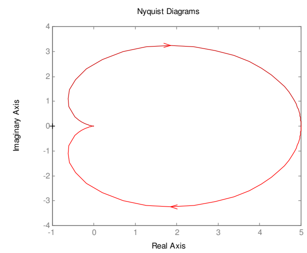


Nyquist path   Nyquist diagram

Examples of Nyquist diagram

- Example 1:

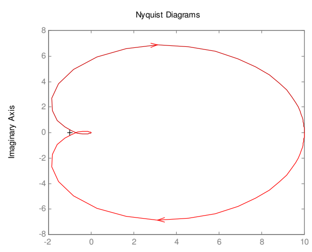
$$G(s)H(s) = \frac{5}{(s+1)^2}$$



Examples of Nyquist diagram

- Example 2:

$$G(s)H(s) = \frac{50}{(s+1)^2(s+5)}$$

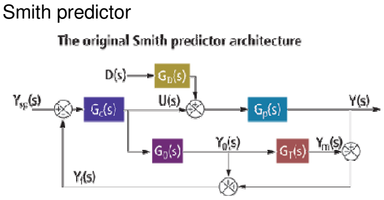


Dealing with time delay

- Time delay is one of the most difficult control problems to overcome
  - Appreciable time delay can cause the controller to become "impatient" (because of the delay, the controller may assume that its initial control efforts had no effect and that it needs to try harder to force a change)
  - Do you still remember the frequency response of a system with an ideal time delay?

Dealing with time delay

- Time delay is one of the most difficult control problems to overcome
- How to deal with time delay?
  - PID control
    - Endow the controller with a degree of patience
    - Throttle back the integral action (why?)
  - Smith predictor



Dealing with time delay

- Time delay is one of the most difficult control problems to overcome
- How to deal with time delay?
- Example 1: Human control of inverted pendulum (with one eye closed)

Dealing with time delay

- Time delay is one of the most difficult control problems to overcome
- How to deal with time delay?
- Example 1: Human control of inverted pendulum
- Example 2: Inverted pendulum with delayed feedback
  - An interesting link:
    http://video.google.com/videoplay?docid=-4853135138529056780&q=inverted+pendulum

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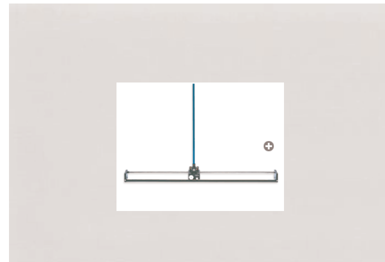

With no time delay

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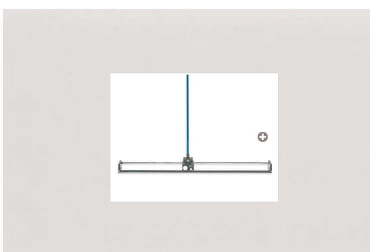

With a time delay of 0.06 sec

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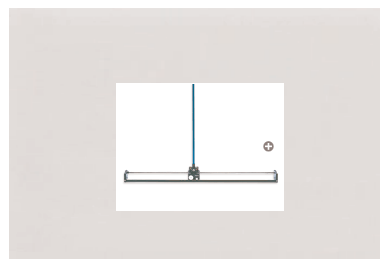

With a time delay of 0.064 sec

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Using adaptive control
(video from Alleyne Research Group, UIUC)

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References

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- http://mr-roboto.me.uiuc.edu/clown/
- http://video.google.com/videoplay?docid=-4853135138529056780&q=inverted+pendulum
- http://www.isa.org/InTechTemplate.cfm?Section=Article_Index1&template=/ContentManagement/ContentDisplay.cfm&ContentID=40670
- http://www.math.uwaterloo.ca/~sacampbe/videos/

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