ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 15: Nyquist Criterion

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Review of last lecture: basic steps for constructing Bode diagrams

- Step 1: Rewrite the transfer function in proper form so that the lowest order term (nonzero) in the numerator and denominator are both unity
- Step 2: Separate the transfer function into its constituent parts—a constant, poles and zeros at the origin, nonzero real poles and zeros, and complex conjugate poles and zeros

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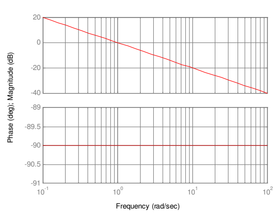
Review of last lecture: poles and zeros at the origin

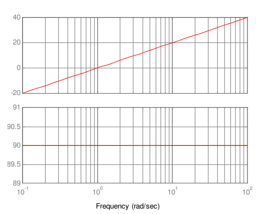
$G(s) = 1/s$

Integrator



$G(s) = s$

Differentiator



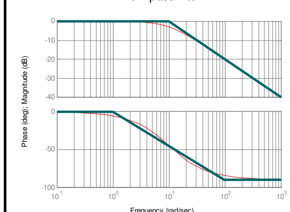
Question 1: What are the slopes of the two magnitude plots?

Question 2: What if we have multiple poles or zeros at the origin?

Review of last lecture: nonzero real poles and zeros

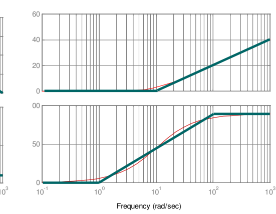
$G(s) = 10/(s+10)$

Low-pass filter



$G(s) = (s+10)/10$

PD controller



Question 1: What are the slopes of the two magnitude plots?

Question 2: What are the break frequencies (or corner frequencies) of the above systems?

Question 3: What if $G(s) = 1-s/10$?

Complex zeros and poles

- Consider poles or zeros of the form

$$s^2 + 2\varsigma\omega_n s + \omega_n^2$$

Question: For which range of the damping ratio do we get complex poles or zeros?

- Straight-line approximations may be very inaccurate for some value of the damping ratio (question: for small value or for large value?)

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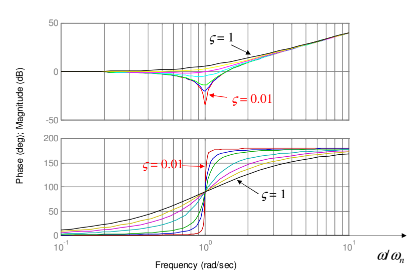
Complex zeros and poles

$$G(s) = 1 + 2\varsigma\frac{s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2$$

Complex zeros



Ideal time delay

Exercise: Sketch the Bode diagram for

$$G(s) = e^{-t_0 s}.$$

Question: Can we approximate the ideal time delay with a rational function?

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Review of last lecture: basic steps for constructing Bode diagrams

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- Step 2: Separate the transfer function into its constituent parts—a constant, poles and zeros at the origin, nonzero real poles and zeros, and complex conjugate poles and zeros
- Step 3: Draw the Bode diagram for each part
- Step 4: Draw the overall Bode diagram by adding up the results from Step 3

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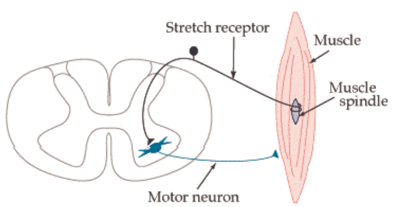
Monosynaptic stretch reflex: “shortest” feedback control in neural systems



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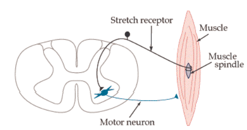
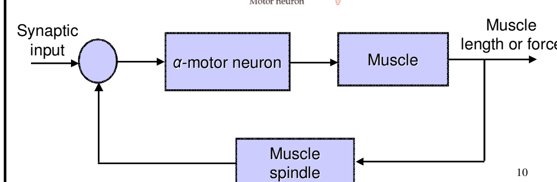
Monosynaptic stretch reflex: “shortest” feedback control in neural systems





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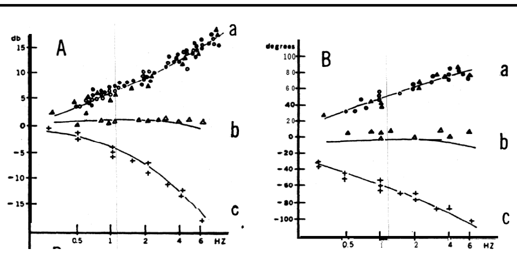

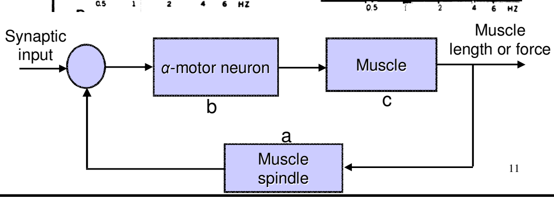

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Outline of this lecture

- What is Nyquist criterion used for?
- Mappings from the complex s-plane to the $F(s)$-plane
- Cauchy’s principle of argument
- Nyquist diagram
- Nyquist criterion
- An example of Nyquist diagram (and Nyquist criterion)

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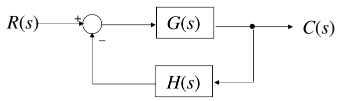
What is Nyquist criterion used for?

- In this lecture we consider the closed-loop systems of the following type



Question 1: What is the characteristic equation of this system?

Question 2: What is the open-loop function of this system?

What is Nyquist criterion used for?

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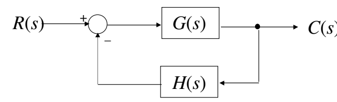

- Nyquist criterion is a unique method for determining stability of a closed-loop system
- Nyquist criterion allows us to determine the stability of a closed-loop system from the Bode diagram (or frequency response) of the open-loop function $G(j\omega)H(j\omega)$

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- Nyquist criterion is a unique method for determining stability of a closed-loop system
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- More about Nyquist: Can anyone name another contribution of Harry Nyquist?

Nyquist rate (Sampling Theorem): For lossless digitization, the sampling rate should be at least twice the maximum frequency responses).

Mappings from the complex s-plane to the $F(s)$-plane

Exercise 1: Considering a function $F(s) = s - s_0$, please map a circle centered at s_0 in the s-plane into the $F(s)$-plane.

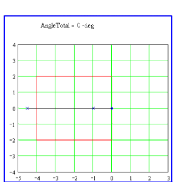
Exercise 2: What if $F(s) = 1/(s - s_0)$?

Question 1: Consider a function $F(s) = (s - s_0)(s - s_1)$ and a curve C (not necessarily a circle) in the s-plane encircling both zeros. As the point s travels around the curve C clockwise, what is the change of angle of $F(s)$? In other words, how many times does $F(s)$ encircle the origin? Clockwise or counterclockwise?

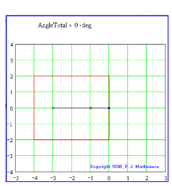
Question 2: What if $F(s) = 1/[(s - s_0)(s - s_1)]$?

Mappings from the complex s-plane to the $F(s)$-plane

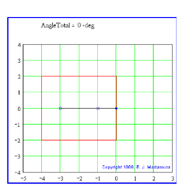
- More examples



Contour 1



Contour 2



Contour 3

Cauchy's principle of argument

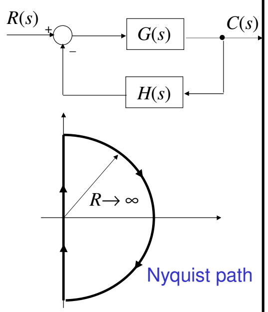
Let $F(s)$ be the ratio of two polynomials in s. Let the closed curve C in the s-plane be mapped into the complex plane through the mapping $F(s)$. If $F(s)$ is analytic (complex differentiable) within and on C, except at a finite number of poles, and if $F(s)$ has neither poles nor zeros on C, then

$$N = Z - P$$

where Z is the number of zeros of $F(s)$ in C, P is the number of poles of $F(s)$ in C, and N is the number of encirclements of the origin, taken in the same sense as C.

Nyquist diagram

Consider a mapping $F(s) = 1 + G(s)H(s)$ and a curve C composed of the imaginary axis and an arc of infinite radius such that the curve completely encloses the right half of the s-plane



Question: For what value of Z (number of zeros of $F(s)$ in C) will the closed-loop system be stable?

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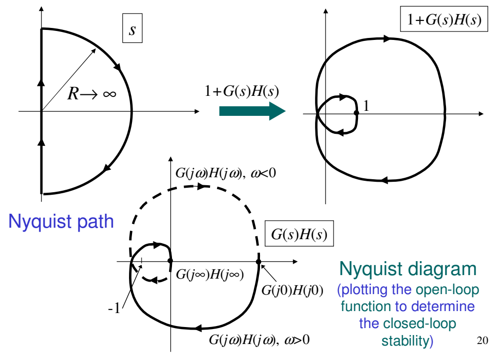
Nyquist diagram



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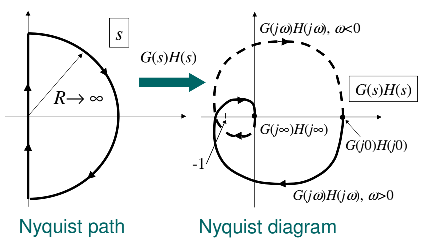
Nyquist criterion



The Nyquist path is shown in the left figure. This path is mapped through the open-loop function $G(s)H(s)$ into the Nyquist diagram, as illustrated in the right figure. Then

$$N = Z - P$$

where Z is the number of roots of the system characteristic equation in the right half-plane, N is the number of clockwise encirclements of the -1 point, and P is the number of poles of the open-loop function $G(s)H(s)$ in the right half-plane.

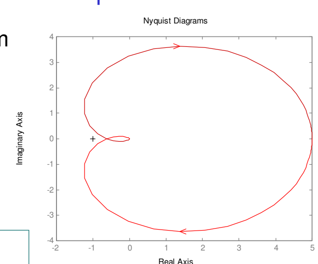
An example

- Consider the system with open-loop function

$$G(s)H(s) = \frac{5}{(s+1)^3}$$

- Matlab program for Nyquist diagram

```
> numGH = [5];
> denGH = [1 3 3 1];
> nyquist(numGH,denGH);
```



Question 1: Is this system stable?

Question 2: What if $G(s)H(s) = \dfrac{5K}{(s+1)^3}$?

References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- http://www.cai.cam.ac.uk/students/study/engineering/engineer03l/cenyquist.htm
- http://www.facstaff.bucknell.edu/mastascu/eControlHTML/CourseIndex.html

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