ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 14: Bode Diagrams Examples

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Homework 6

- Problem 8.6 (a)-(e) in the text book
- Due next Wednesday

About last lecture

- Three steps for general design of controllers
- Advantages and disadvantages of root-locus design
- Three ways of visualizing frequency-response function
- Definition of Bode diagram
  - Bode diagram, or Bode plot, is a plot of magnitude versus frequency and phase versus frequency with logarithmic frequency scale and magnitude scale
- Main advantage of Bode diagram

Outline of this lecture

- Approximate sketches of Bode diagrams
- Bode diagrams examples
  - Constant gain
  - Integrators and differentiators (poles and zeros at the origin)
  - Nonzero real poles and zeros
  - Complex poles and zeros
  - Ideal time delay
- Basic steps for constructing Bode diagrams

Approximate sketches of Bode diagrams

- Approximate magnitude diagram for a first-order frequency-dependent term

$$\boxed{1+j\omega/\omega_i.}$$

  - Its magnitude in decibels equals

$$\boxed{\mathrm{dB}_i = 20\log[1+(\omega/\omega_i)^2]^{1/2}.}$$

Why?

$$\boxed{\text{For } \omega << \omega_i,\ \mathrm{dB}_i \approx 0}$$

Why?

$$\boxed{\text{For } \omega >> \omega_i,\ \mathrm{dB}_i \approx 20\log\omega - 20\log\omega_i}$$

Why?

Approximate sketches of Bode diagrams

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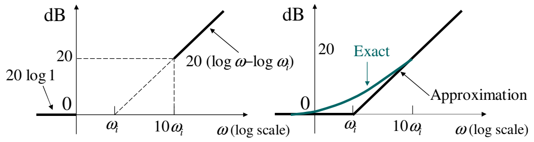


Approximate sketches of Bode diagrams

- Approximate magnitude diagram for a first-order frequency-dependent term
- Approximate phase diagram for a first-order frequency-dependent term

$1 + j\omega/\omega_i.$ Its phase angle equals

$$\theta(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_i}\right).$$

For $\omega << \omega_i,\ \theta(\omega) \approx 0$

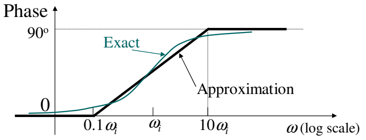
For $\omega >> \omega_i,\ \theta(\omega) \approx 90^{o}$

Approximate sketches of Bode diagrams

- Approximate magnitude diagram for a first-order frequency-dependent term
- Approximate phase diagram for a first-order frequency-dependent term

$1 + j\omega/\omega_i.$ $\theta(\omega) = \tan^{-1}(\omega/\omega_i).$

For $\omega << \omega_i,\ \theta(\omega) \approx 0$ For $\omega >> \omega_i,\ \theta(\omega) \approx 90^{o}$

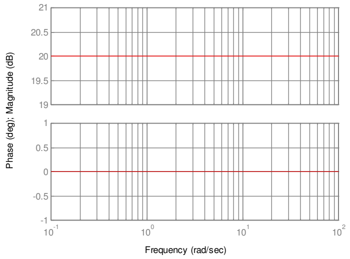


Constant gain

Exercise 1: Sketch the Bode diagram for $G(s) = K$.

Constant gain

Constant gain K=10



Questions: For what range of K does the system have positive (or negative) dB? What does the Bode diagram look like if $K<0$?

Poles and zeros at the origin

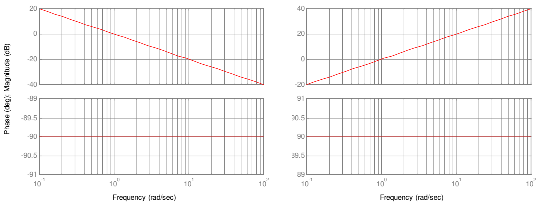
Exercise 1: Sketch the Bode diagram for $G(s) = 1/s$.

Exercise 2: Sketch the Bode diagram for $G(s) = s$.

Poles and zeros at the origin

$G(s) = 1/s$ Integrator

$G(s) = s$ Differentiator



Question 1: What are the slopes and zero-dB frequencies of the two magnitude plots?

Question 2: What if we have multiples poles or zeros at the origin?

Nonzero real poles and zeros

Exercise 1: Sketch the Bode diagram for $G(s) = 10/(s+10)$.

Exercise 2: Sketch the Bode diagram for $G(s) = (s+10)/10$.

Question: What are the break frequencies (or corner frequencies) of the above systems?

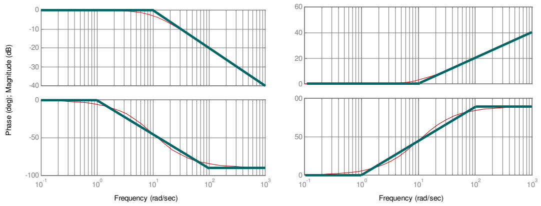
Nonzero real poles and zeros

$G(s) = 10/(s+10)$

Low-pass filter

$G(s) = (s+10)/10$

PD controller



Question 1: What are the slopes of the two magnitude plots?

Question 2: What are the limits of the phase angles as $\omega \rightarrow \infty$?

Nonzero real poles and zeros

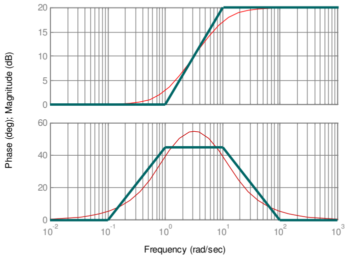
Exercise 3: Sketch the Bode diagram for $G(s) = 10(s+1)/(s+10)$.

Question: Is this a phase-lead controller or phase-lag controller?

Nonzero real poles and zeros

$$G(s) = \frac{10(s+1)}{s+10}$$

Phase-lead controller



Complex zeros and poles

- Consider poles or zeros of the form

$$s^2 + 2\varsigma\omega_n s + \omega_n^2.$$

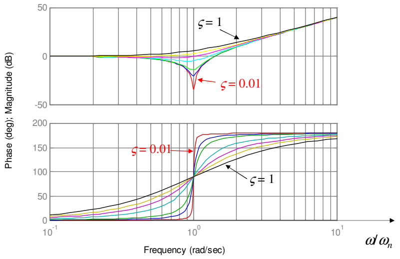
Question: For which range of the damping ratio do we get complex poles or zeros?

- Straight-line approximations may be very inaccurate for some value of the damping ratio (question: for small value or for large value?)

Complex zeros and poles

$$G(s) = 1 + 2\varsigma\frac{s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2$$

Complex zeros



Ideal time delay

Exercise: Sketch the Bode diagram for

$$G(s) = e^{-t_0 s}.$$

Question: Can we approximate the ideal time delay with a rational function?

Basic steps for constructing Bode diagrams

- Step 1: Rewrite the transfer function in proper form so that the lowest order term (nonzero) in the numerator and denominator are both unity
  - Examples:

$$G(s) = \frac{30(s+10)}{s^2+3s+50} = \frac{30\cdot 10\cdot(\frac{s}{10}+1)}{50\cdot(\frac{s^2}{50}+\frac{3}{50}s+1)} = 6\frac{\frac{s}{10}+1}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

$$G(s) = \frac{5s}{s^2+3s+50} = \frac{5\cdot\frac{s}{1}}{50\cdot(\frac{s^2}{50}+\frac{3}{50}s+1)} = \frac{1}{10}\frac{\frac{s}{1}}{\frac{s^2}{50}+\frac{3}{50}s+1}$$

Basic steps for constructing Bode diagrams

- Step 1: Rewrite the transfer function in proper form so that the lowest order term (nonzero) in the numerator and denominator are both unity
- Step 2: Separate the transfer function into its constituent parts—a constant, poles and zeros at the origin, nonzero real poles and zeros, and complex conjugate poles and zeros

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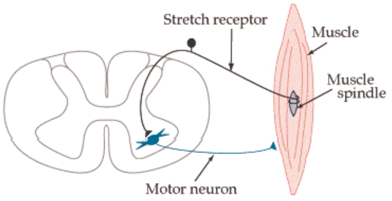
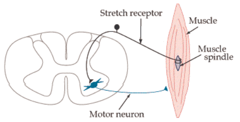
Basic steps for constructing Bode diagrams

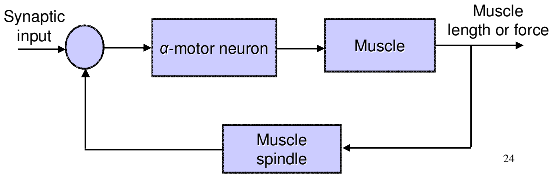
- Step 1: Rewrite the transfer function in proper form so that the lowest order term (nonzero) in the numerator and denominator are both unity
- Step 2: Separate the transfer function into its constituent parts—a constant, poles and zeros at the origin, nonzero real poles and zeros, and complex conjugate poles and zeros
- Step 3: Draw the Bode diagram for each part
- Step 4: Draw the overall Bode diagram by adding up the results from Step 3

Monosynaptic stretch reflex: “shortest” feedback control in neural systems



Monosynaptic stretch reflex: “shortest” feedback control in neural systems





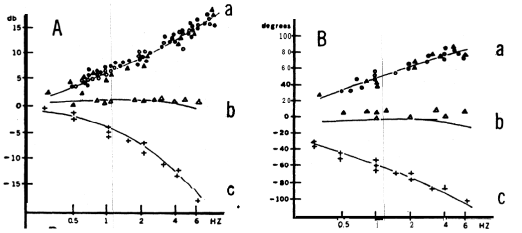
Synaptic input

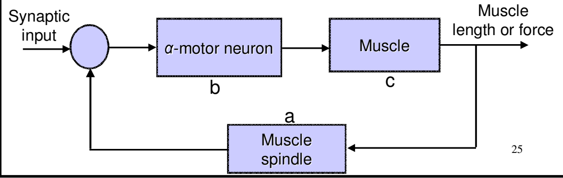
α-motor neuron

Muscle

Muscle length or force

Muscle spindle





Synaptic input

α-motor neuron

b

Muscle

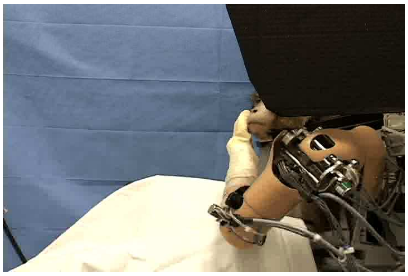
c

Muscle length or force

a

Muscle spindle

For those of you who are interested in neural control and neuroengineering



References

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- http://motorlab.neurobio.pitt.edu/multimedia.php