ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 13: More on Root-Locus Design; Frequency Response Analysis; Bode Diagrams

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Instructor: Zhi-Hong Mao
Assistant Professor of ECE and Bioengineering
University of Pittsburgh, Pittsburgh, PA

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About last lecture

- General first-order compensators

$$G_c(s) = \frac{K_c(s - z_0)}{(s - p_0)}$$

- Phase-lead compensator

Question 1: Why do we use phase-lead compensator?

- Phase-lag compensator

Question 2: What is the function of phase-lag compensator?

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About last lecture

- General first-order compensators
- Phase-lead compensator
- Phase-lag compensator
- PID controller

$$G_c(s) = K_P + \frac{K_I}{s} + K_D s$$

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
K_P	?	?	?	?
K_I	?	?	?	?
K_D	?	?	?	?

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About last lecture

- General first-order compensators
- Phase-lead compensator
- Phase-lag compensator
- PID controller
  - P control

$$G_c(s) = K_P \qquad G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
K_P	Decrease	Increase	Small change	Decrease
K_I	?	?	?	?
K_D	?	?	?	?

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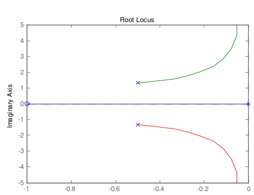
About last lecture



- General first-order compensators
- Phase-lead compensator
- Phase-lag compensator
- PID controller
  - P control
  - I control (usually we use PI instead of using I alone)

$$G_c(s) = K_P + \frac{K_I}{s} \qquad G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
K_P	Decrease	Increase	Small change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	?	?	?	?

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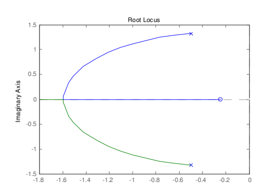
About last lecture



- General first-order compensators
- Phase-lead compensator
- Phase-lag compensator
- PID controller
  - P control
  - I control
  - D control (usually we use PD instead of using D alone)

$$G_c(s) = K_P + K_D s \qquad G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
K_P	Decrease	Increase	Small change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small change	Decrease	Decrease	Small change

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About last lecture

- General first-order compensators
- Phase-lead compensator
- Phase-lag compensator
- PID controller
  - P control
  - I control
  - D control
  - Another view on PID control (based on information from the past, present, and future)

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An example of temperature control from hospital

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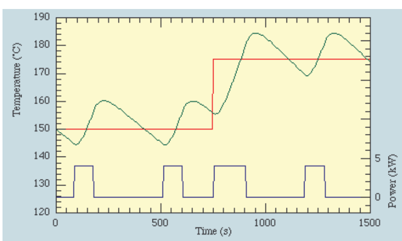
Temperature control using feedback



On-off control

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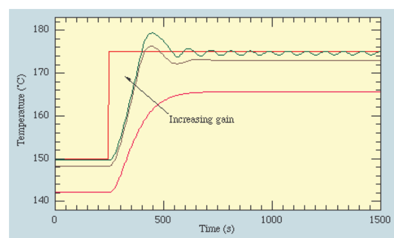
Temperature control using feedback



Proportional control

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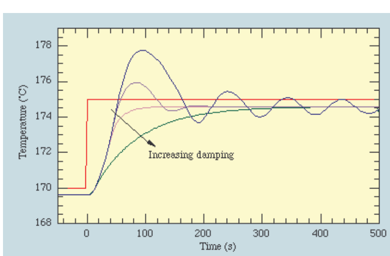
Temperature control using feedback



PD control

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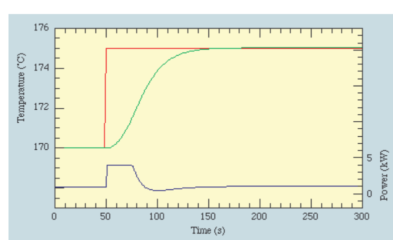
Temperature control using feedback



PID control

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Outline of this lecture

- Brief review of root-locus design
- Frequency-response methods as a good complement to the root-locus techniques
- Frequency-responses and frequency response function
- Different ways of visualizing frequency-response function
- Bode diagrams

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Brief review of root-locus design

- Three steps for general design of controllers
  - Determine what the controller should do (design specifications)

Question 1: Can you name some design specifications?

Question 2: What design specifications can root-locus methods handle?

Hint: Relative stability, steady-state accuracy, transient response (maximum overshoot, rise time, and settling time), and frequency-response characteristics, robustness (sensitivity to parameter variations, disturbance rejection)

  - Determine the controller or compensator configuration relative to how it is connected to the controlled process
  - Determine the parameter values of the controller to achieve the design goals

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Brief review of root-locus design

- Three steps for general design of controllers
- Root-locus methods allow us to control the transient response to some extent (why?)

Hint: Closed-loop system pole locations; modes of transient response

- Root-locus methods can also influence the steady-state response (how?)

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Brief review of root-locus design

- Three steps for general design of controllers
- Root-locus methods allow us to control the transient response to some extent
- Root-locus methods can also influence the steady-state response
- Advantages:
  - Have good indication of the transient response of a system
  - Explicitly show location of all closed-loop poles
- Disadvantage:
  - Need a relatively accurate model of the system in root-locus design
  - Hard to determine response in steady-state

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Frequency-response methods as a good complement to the root-locus techniques

- Frequency-response methods can provide good design in the face of uncertainty in the plant model (we do not need an accurate transfer function)
- Experimental information can be easily used for design purposes

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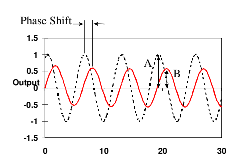
Frequency response and frequency response function

- Frequency response: steady-state response of systems to sinusoidal inputs

The figure compares the output response of a system (red solid line) with a sinusoidal input (black dashed line)

Both the magnitude and the phase shift of a system will change with the frequency of the input into the system



Amplitude Ratio = B / A

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Frequency response and frequency response function

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response function

Question: Given a system with transfer function $G(s)$, what is its frequency response function?

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Frequency response and frequency response function

- Frequency response: steady-state response of systems to sinusoidal inputs
- Frequency response function
  - Given a system with transfer function $G(s)$, its frequency response function is $G(j\omega)$
  - The steady-state gain of a system for a sinusoidal input is the magnitude of the transfer function evaluation at $s = j\omega_1$, and the phase shift of the output sinusoid relative to the input sinusoid is the angle of $G(j\omega_1)$

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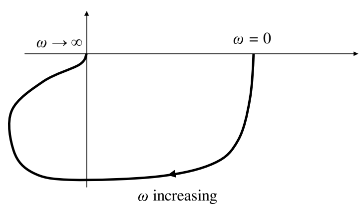
Different ways of visualizing frequency-response function

- Method 1: Polar plot
  - The frequency response is a mapping from the s-plane (which part?) to the $G(j\omega)$-plane



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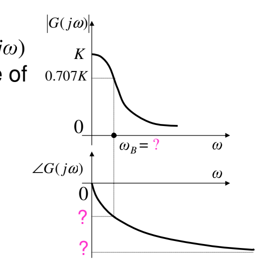
Different ways of visualizing frequency-response function

- Method 1: Polar plot
- Method 2: Plot the magnitude (gain) of $G(j\omega)$ versus ω and the angle of $G(j\omega)$ versus ω
  - Example 1:

$$G(s) = \frac{K}{\tau s + 1}$$



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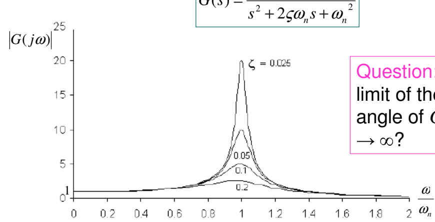
Different ways of visualizing frequency-response function

- Method 1: Polar plot
- Method 2: Plot the magnitude (gain) of $G(j\omega)$ versus ω and to plot the angle of $G(j\omega)$ versus ω
  - Example 2:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$



Question: What is the limit of the phase angle of $G(j\omega)$ as $\omega \to \infty$?

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Different ways of visualizing frequency-response function

- Method 1: Polar plot
- Method 2: Plot the magnitude (gain) of $G(j\omega)$ versus ω and to plot the angle of $G(j\omega)$ versus ω
- Method 3: Bode diagrams

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Bode diagrams

- Definition:
  - Bode diagram, or Bode plot, is a plot of magnitude versus frequency and phase versus frequency with logarithmic frequency scale and magnitude scale
  - We plot the magnitude of the frequency response in decibels (dB)

$$\text{dB} = 20\ \log\ [\text{magnitude}]$$

Question: Which magnitude corresponds to 20 dB?

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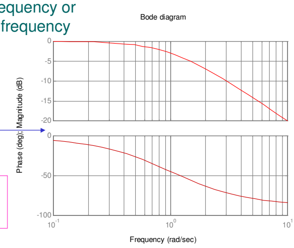
Bode diagrams

- Definition:
  - Bode diagram, or Bode plot, is a plot of magnitude versus frequency and phase versus frequency with logarithmic frequency scale and magnitude scale
  - We plot the magnitude of the frequency response in decibels (dB)
  - Example:

Break frequency or corner frequency

$$G(s) = \frac{K}{1+\tau_1 s} = \frac{K}{1+s/\omega_1}$$

Right figure shows the Bode diagram of $G(s)$ for $K = 1$ and $\omega_1 = 1$



Question: How does the Bode diagram change if we change the value of K or ω_1 ?

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Bode diagrams

- Definition
- Main advantage of the Bode diagram
  - The effects of adding a real pole or a real zero to a transfer function can be seen rather easily (for this reason, Bode diagrams are very useful in designing control systems)

Question: Can we (and if we can, how can we) draw the Bode diagram for $G_1(s)\ G_2(s)$ based on the Bode diagrams of $G_1(s)$ and $G_2(s)$?

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Bode diagrams

- Definition:
- Main advantage of the Bode diagram
- Using Matlab for Bode plot

```
> numG = [1 0 2 3];       % Define numerator of G(s).
> denG = [1 1 1 1 1];     % Define denominator of G(s).
> G = tf(numG,denG)       % Create and display G(s).
> bode(G);                % Draw Bode diagram. Or use
                          %       bode(numG,denG);
```

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Bode diagrams

- Definition:
- Main advantage of the Bode diagram
- Using Matlab for Bode plot
- Example: Pilot-vehicle system dynamics

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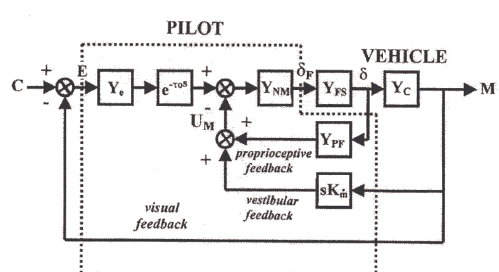

Pilot-vehicle system

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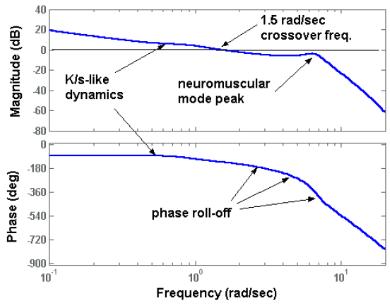

Bode diagram of open loop pilot-vehicle transfer function (with tracking error as input and tracking angle as output)

Mao in a driving test

References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- G. C. Goodwin, S. F. Graebe, and M. E. Salgado. Control System Design. Prentice Hall, 2000.
- http://newton.ex.ac.uk/teaching/CDHW/Feedback/