ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 12: Phase-Lead and Phase-Lag Compensators; PID Control

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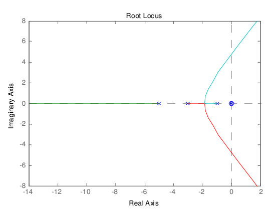
Homework 5 and Lab 2

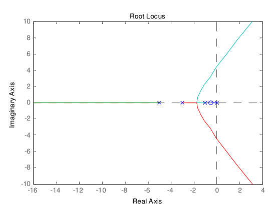
- Homework 5
  - Problems 7.8 and 8.1 (a)-(d) in the text book
  - Due next Wednesday 5:00 pm

$$KGH = \frac{K(s+a)}{s(s+1)(s+3)(s+5)}$$

- Lab 2
  - How will the root-locus plot change if the value of a changes?
    - The asymptotes (angles and intersecting point)
    - Dominant poles
    - Stability margin

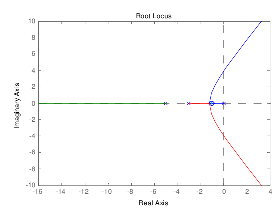


$a = 0$

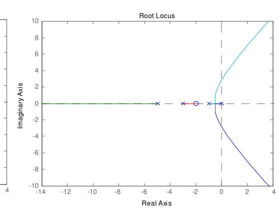


$a = 0.5$

$$KGH = \frac{K(s+a)}{s(s+1)(s+3)(s+5)}$$

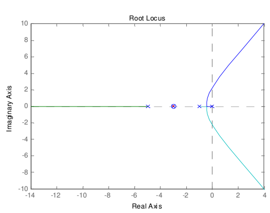


$a = 1$

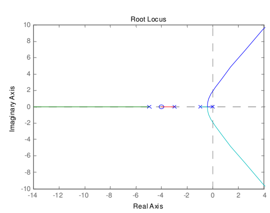


$a = 2$

$$KGH = \frac{K(s+a)}{s(s+1)(s+3)(s+5)}$$

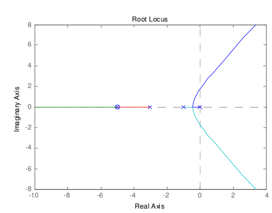


$a = 3$

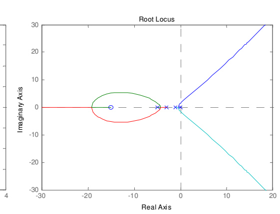


$a = 4$

$$KGH = \frac{K(s+a)}{s(s+1)(s+3)(s+5)}$$



$a = 5$



$a = 15$

$$KGH = \frac{K(s+a)}{s(s+1)(s+3)(s+5)}$$

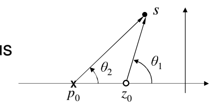
Outline of this lecture

- General first-order compensators
- Phase-lead compensator
- Phase-lag compensator
- PID control

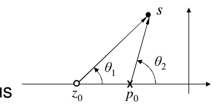
General first-order compensators

- Phase-lead compensator
  - $|z_0| < |p_0|$ (the zero is closer to the origin than the pole)
  - Positive contribution to the angle criterion of the root locus

$$G_c(s) = \frac{K_c(s - z_0)}{(s - p_0)}$$

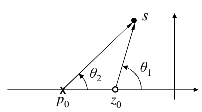


- Phase-lag compensator
  - $|z_0| > |p_0|$ (the pole is closer to the origin than the zero)
  - Negative contribution to the angle criterion of the root locus



Phase-lead compensator

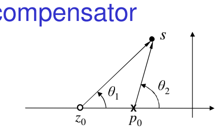
$$G_c(s) = \frac{K_c(s - z_0)}{(s - p_0)}$$



- The positive angle contributed by the phase-lead compensator will tend to shift the root locus of the plant toward the left in the s-plane, that is, toward the more stable region

Phase-lag compensator

$$G_c(s) = \frac{K_c(s - z_0)}{(s - p_0)}$$



- Phase-lag compensator and system stability
  - The phase-lag compensator will tend to shift the root locus of the plant toward the right in the s-plane, that is, toward the more unstable region
  - In design, the angle contribution of the phase-lag compensator must be small, to minimize the destabilizing effects
  - The pole and zero of phase-lag compensator should be placed very close to each other

Phase-lag compensator

- Phase-lag compensator and system stability
- The Phase-lag compensator is used to improve the system steady-state response. Reasoning:
  - Assume that the compensator has unity dc gain

$$G_c(s) = \left.\frac{K_c(s - z_0)}{(s - p_0)}\right|_{s=0} = \frac{K_c z_0}{p_0} = 1$$

  - Assume that the root locus of the uncompensated system passes through the value s_1 for gain K_0 and that this point on the root locus gives a satisfactory transient response

$$1 + K_0 G_p(s_1)H(s_1) = 0 \;\Rightarrow\; K_0 = \frac{-1}{G_p(s_1)H(s_1)}$$

  - Choose the values of z_0 and p_0 to be approximately equal, and choose the magnitudes of z_0 and p_0 small compared to $|s_1|$

Question: What does this imply about the system steady-state response?

$$K = \frac{-1}{G_c(s_1)G_p(s_1)H(s_1)} \overset{?}{\approx} \frac{-1}{K_c G_p(s_1)H(s_1)} = \frac{K_0}{K_c}$$

Phase-lag compensator

- Phase-lag compensator and system stability
- The Phase-lag compensator is used to improve the system steady-state response
- Steps in designing a phase-lag compensator
  - Choose K_0 (see previous slide) to yield the value of the desired closed-loop pole s_1 in the uncompensated system
  - Calculate the value of K required to yield the desired steady-state response, assuming that the compensator dc gain is unity. Then calculate $K_c = K_0 / K$
  - Choose the magnitude of the compensator zero, $|z_0|$, small compared to $|s_1|$
  - Calculate the compensator pole: $p_0 = K_c z_0$

PID control

- Proportional control
  - In proportional control, steady-state error tends to depend inversely upon proportional gain
  - Proportional control has a tendency to make a system faster
  - Proportional control does not change the order of the system

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PID control

- Proportional control
- Integral control
  - In integral control, steady-state error should be zero (prerequisite: the closed loop system has to be stable)
  - Integral control has a tendency to make a system slower and may even sacrifice stability (why?)

Hint: A pole at origin.

  - Integral control changes the order of the system

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PID control

- Proportional control
- Integral control
- Derivative control
  - Derivative control tends to increase the stability of the system
  - Derivative control tends to reduce the overshoot and improve the transient response
  - Derivative control changes the order of the system

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PID control

- Proportional control
- Integral control
- Derivative control

Closed-loop response	Rise time	Overshoot	Settling time	Steady-state error
K_P	Decrease	Increase	Small change	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	Small change	Decrease	Decrease	Small change

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PID control

- Proportional control
- Integral control
- Derivative control
- Another view on PID control
  - The proportional term gives the controller output a component that is a function of the present state of the system
  - The integrator output is determined by the past state of the system
  - The differentiator is a function of the slope of its input and thus can be considered to be a predictor of the future state of the system
  - The PID controller can viewed as giving control that is a function of the past, the present, and the predicted future

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PID control

- Proportional control
- Integral control
- Derivative control
- Another view on PID control
- An example of PID controller in neural systems: cerebellum

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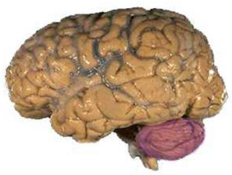
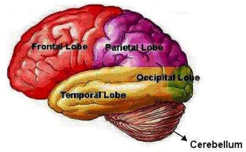
An example of PID controller in neural systems: cerebellum





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Intention tremor

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References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- G. C. Goodwin, S. F. Graebe, and M. E. Salgado. Control System Design. Prentice Hall, 2000.
- http://www.aafp.org/afp/20031015/1545.html

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