ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 11: Root Locus Design and Practice Problems for Quiz 1

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About Lab 3

- Notes available online
- Due March 18
- Supplemental note for Lab 3 (also see course webpage)
  - Ziegler-Nichols oscillation method
  - Ziegler-Nichols reaction curve method
  - Cohen-Coon reaction curve method

Questions from Homework 3

- When should we worry about high frequencies?
  - Steady-state error
  - Stability
  - Differentiator
  - Integrator
- Different behaviors of unstable LTI systems
  - What about nonlinear systems?

Six Rules for root locus plot

- The root locus is symmetrical with respect to the real axis
- The root locus originates on the poles of $G(s)H(s)$ (for $K = 0$) and terminates on the zeros of $G(s)H(s)$ (as $K \to \infty$), including those zeros at infinity
- If the open-loop function has zeros at infinity, the root locus approaches asymptotes as K approaches infinity. The asymptotes are located at the angles $\theta = r180^\circ / \alpha, \quad \alpha = n - m, \quad r = \pm 1, \pm 3, \ldots$
  and these asymptotes intersect the real axis at the point
  $$\sigma_a = \frac{\text{(sum of finite poles) - (sum of finite zeros)}}{\text{(number of finite poles) - (number of finite zeros)}}$$
- The root locus includes all points on the real axis to the left of an odd number of real critical frequencies (poles and zeros)
- The breakaway points on a root locus will appear among the roots of the polynomial obtained from $N(s)D'(s) - N'(s)D(s) = 0,$
  where $N(s)$ and $D(s)$ are the numerator and denominator polynomials, respectively, of $G(s)H(s)$
- Loci will depart from a pole p_j (arrive at a zero z_j) of $G(s)H(s)$ at the angle θ_d (θ_a), where $\theta_d = \sum_i \theta_{zi} - \sum_{i, i \neq j} \theta_{pi} + r(180^\circ),\ \theta_a = \sum_i \theta_{pi} - \sum_{i, i \neq j} \theta_{zi} + r(180^\circ)$
  and where $r = \pm 1, \pm 3, \ldots$ and θ_{pi} (θ_{zi}) represent the angles from pole p_i (zero z_i), respectively, to p_j (zero z_j)

Exercise: Plot root locus for

$$G(s)H(s) = \frac{s}{s^2 + 5}$$

Outline of this lecture

- Root locus design
  - An example: attitude control for a rigid satellite
- Practice problems for Quiz I

Attitude control for a rigid satellite

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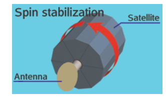
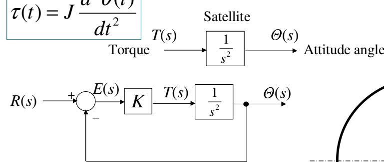
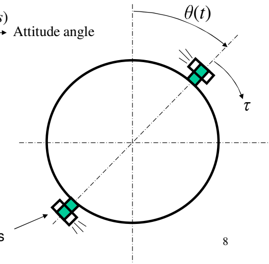
Attitude control for a rigid satellite



$$\tau(t) = J\frac{d^2\theta(t)}{dt^2}$$





Exercise: Plot the root locus

Question: For what value of K will the closed-loop system be stable?

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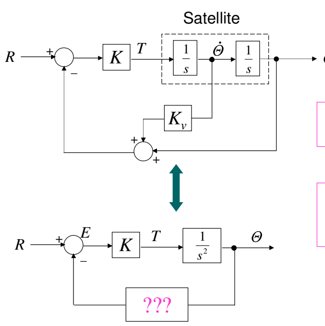
Attitude control for a rigid satellite



Exercise: Plot the root locus with respect to K.

Question: For what value of K will the closed-loop system be stable?

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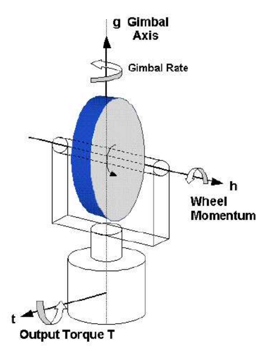
Attitude control for a rigid satellite

Question: Besides thrustors, what else can be used for torque generation?



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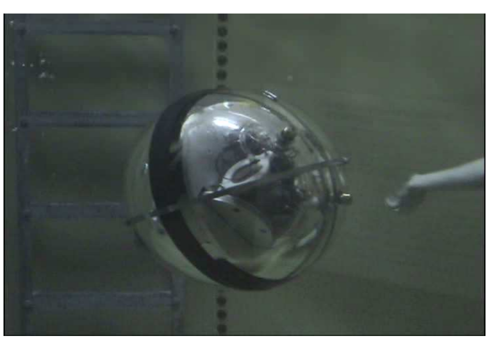

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Practice problems for Quiz 1

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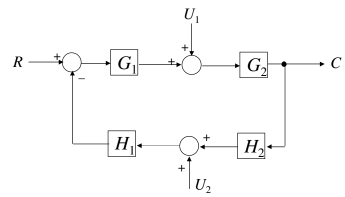
Problem 1: Partial fraction expansion

$$F(s) = \frac{s^2 + 2s + 2}{s^2 + 3s + 2}$$

Problem 2: Block diagram and transfer function



Determine the output C due to U_1, U_2, and R.

Problem 3: System sensitivity

$$T = \frac{A_1 + kA_2}{A_3 + kA_4}$$

Determine the sensitivity of T with respect to k.

Problem 4: Root locus plot

$$KG(s)H(s) = \frac{K}{s(s^2 + 20s + 101)}$$

References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- http://heasarc.gsfc.nasa.gov/Videos/XTE/