ECE 1673: Linear Control Systems (4 Credits, Spring 2009)

Lecture 10: Root Locus Examples

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Homework 4

- Problem 7.7 in the text book
  - This problem will not be turned in or graded
  - Please verify your results with Matlab
  - I suggest you to finish this problem before the quiz

A question

- Which part of the loop should we look at?
  - Stability
  - System type
  - Root locus plot

Study guide for Quiz I

- Laplace transform and inverse Laplace transform
  - Partial fraction expansion of a rational function (won't cover the case of repeated roots with an order of $r > 2$)
  - Final value theorem
  - Differential theorem (note: 0^- not 0^+)
  - Shift in time domain and frequency domain
- Transfer function
  - Finding transfer function in a block diagram
  - Won't cover the signal flow graphs and Mason's gain formula

Study guide for Quiz I

- Responses of first and second order systems
  - Impulse, step, ramp, and frequency responses of first order systems; concepts of time constant, dc gain, and system bandwidth
  - Poles of a second order systems; concepts of damping ratio and natural frequency
  - Frequency response function $G(j\omega)$
- Calculation of sensitivity functions
- Stability
  - Criteria for BIBO stability of LTI systems
  - Won't cover Routh criterion

Study guide for Quiz I

- Steady-state accuracy in a system with type $N < 2$
- Root locus plots
  - The six rules for root locus plots
  - Won't cover the phase lead or phase lag compensators or PID controller design (these will be covered by Quiz II)

Review of last lecture

- Angle criterion

$\angle KG(s)H(s) = r(180^\circ),$ where $r = \pm 1, \pm 3, \ldots$ $\overset{?}{\Leftrightarrow}$ $\angle G(s)H(s) = r(180^\circ),$ where $r = \pm 1, \pm 3, \ldots$

Question 1: Given that s_1 satisfies the angle criterion, is it always true that s_1 is on the root locus?

- Magnitude criterion $|KG(s)H(s)| = 1$

Question 2: Given that s_1 satisfies the magnitude criterion, is it always true that s_1 is on the root locus?

- Axis crossings $1 + KG(j\omega)H(j\omega) = 0$
- Six rules for root locus plot

Six Rules for root locus plot

- The root locus is symmetrical with respect to the real axis
- The root locus originates on the poles of $G(s)H(s)$ (for $K = 0$) and terminates on the zeros of $G(s)H(s)$ (as $K \to \infty$), including those zeros at infinity
- If the open-loop function has zeros at infinity, the root locus approaches asymptotes as K approaches infinity. The asymptotes are located at the angles $\theta = r180^\circ/\alpha, \quad \alpha = n - m, \quad r = \pm 1, \pm 3, \ldots$ and these asymptotes intersect the real axis at the point

$$\sigma_a = \frac{\text{(sum of finite poles) - (sum of finite zeros)}}{\text{(number of finite poles) - (number of finite zeros)}}$$

- The root locus includes all points on the real axis to the left of an odd number of real critical frequencies (poles and zeros)
- The breakaway points on a root locus will appear among the roots of the polynomial obtained from $N(s)D'(s) - N'(s)D(s) = 0,$ where $N(s)$ and $D(s)$ are the numerator and denominator polynomials, respectively, of $G(s)H(s)$
- Loci will depart from a pole p_j (arrive at a zero z_j) of $G(s)H(s)$ at the angle θ_d (θ_a), where $\theta_d = \sum_{i=1}^{n} \theta_{zi} - \sum_{\substack{i=1 \\ i \neq j}}^{m} \theta_{pi} + r(180^\circ),\ \theta_a = \sum_{i=1}^{m} \theta_{pi} - \sum_{\substack{i=1 \\ i \neq j}}^{n} \theta_{zi} + r(180^\circ)$ and where $r = \pm 1, \pm 3, \ldots$ and θ_{pi} (θ_{zi}) represent the angles from pole p_i (zero z_i), respectively, to p_j (zero z_j)

Outline of this lecture

- Root locus examples

Example 1

$$G(s)H(s) = \frac{1}{(s-1)(s+2)(s+3)} = \frac{1}{s^3 + 4s^2 + s - 6}$$

Example 2

$$G(s)H(s) = \frac{s+1}{s^2}$$

Question: What is its system type?

Example 3

$$G(s)H(s) = \frac{s+2}{(s+1)(s+3)}$$

Example 4

$$G(s)H(s) = \frac{s+3}{(s+1)(s+2)}$$

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Example 5

$$G(s)H(s) = \frac{1}{s(s^2+8s+32)}$$

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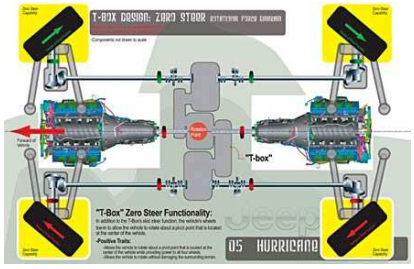
About Jeep Hurricane Concept



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About Jeep Hurricane Concept

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Tracked vehicles vs wheeled vehicles

Study Results	Tracked Vehicles	Wheeled Vehicles
Route Flexibility	✓	
Cross Country Mobility	✓	
Traction on Slopes	✓	
Road Speed		✓
Logistics		✓
O&S Costs		✓
GVW, Volume, & Payload	✓	
Maneuverability/Turning Radius	✓	
Transportability	✓	
Weight Growth Potential	✓	
Gap & Obstacle Crossing	✓	

Results from U.S. Army investigation as of 1998. O & S: Operating and Support: GVW: Gross Vehicle Weight

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References

- C. L. Phillips and R. D. Harbor. Feedback Control Systems, 4th Edition, Prentice Hall, 2000.
- http://trucks.about.com/library/photo/bl_jeep_d5_10.htm

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