ECE 1673: Linear Control Systems

Lab 2: Analysis of Control System Stability

(Provided by Dr. Ian C. Bruce, McMaster University)

Objectives:

To gain experience in analyzing the stability of a closed-loop feedback control system within MATLAB/Simulink.

Assessment:

Your grade for this lab will be based on your ability to (1) apply various mathematical algorithms, (2) write MATLAB code, and (3) create a Simulink model to investigate the stability of the control system described below and on your reporting of the results. The report should contain the mathematical derivations and calculations carried out, MATLAB plots of results, a schematic of your Simulink model, and answers to specific questions below. Each team should complete one lab report together.

Description of tracked vehicle turning control:

The design of a turning control for a tracked vehicle involves the selection of two parameters. In Fig. 1, the system shown in panel (a) has the model shown in panel (b). The parameters a and K affect the performance of the system, including its stability.

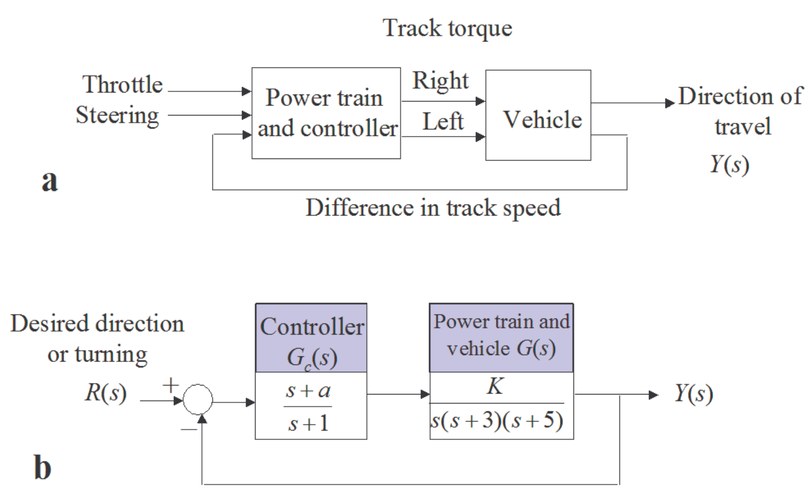

Figure 1. Tracked vehicle control system. **a**. Turning control system for a two-track vehicle. **b**. Linear system model.

1. Routh's criterion for internal stability

a. Determine the closed-loop characteristic polynomial for the model shown in Fig. 1(b).

b. Generate Routh's array for this characteristic polynomial.

c. Determine under what conditions (i.e., values of a and K) the closed-loop system is internally stable.

d. In MATLAB, plot the curve of a versus K (for $K > 0$) that divides the regions of stability and instability, and indicate on the plot which is the stable region.

2. Steady state error to a ramp reference

For a unity-gain closed-loop feedback system with open-loop transfer function $G_c(s)G(s)$, reference $r(t)$, output $y(t)$ and error $e(t) = r(t) - y(t)$, the steady-state error is

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} \frac{sR(s)}{1+G_c(s)G(s)}$$

a. Derive the steady-state error e_{ss} for a ramp reference $r(t) = At$, $t > 0$.

b. Determine under what conditions (i.e., values of a and K) the steady-state error is less than or equal to 22% of the ramp slope A.

3. Root-locus analysis

In MATLAB, generate the root-locus plot for this system, for values of K between 0 and some large number approximating ∞. Use the same value of a as above. [*Hint* 1: Use the MATLAB function `roots()` to find the roots of the characteristic polynomial.]

On the root-locus plot, indicate the starting point and ending point (if it is finite) of each root locus. What does each of these correspond to in the open-loop transfer function?

4. Simulation of the model in Simulink

Create a Simulink model of the control system with a ramp reference as described in Section 2 above.

a. Choose some value for a between 0 and 1. Now choose some value for K such that, given your choice for a, the system is stable and has a steady-state error less than or equal to 22% of the reference ramp slope A. Plot the ramp reference $r(t)$ and the model output $y(t)$ versus t—try this for a couple of different values of A.

b. Leaving the value of a the same as above, increase the value of K such that the system should be unstable. Plot the ramp reference $r(t)$ and the model output $y(t)$ versus t and confirm that the system is indeed unstable with these parameters.