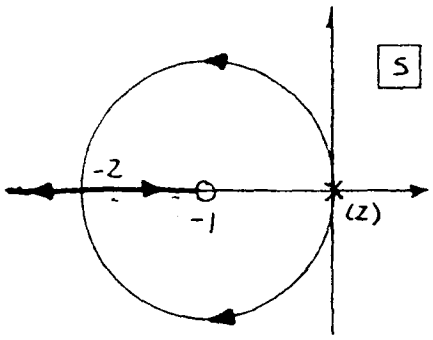
7.7. (a) $KG(s) = \dfrac{K(s+1)}{s^2}$

$ND' - DN' = 0$

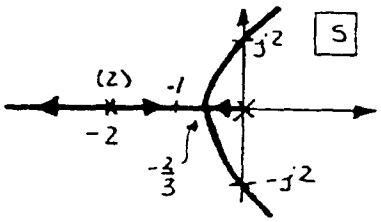
$(s+1)2s - s^2(1) = 0$

$s^2 + 2s = s(s+2) = 0$

$\therefore s = 0, -2$



(b) $KG(s) = \dfrac{K}{s(s+2)^2}$



$\sigma_a = \dfrac{-4+0}{3-0} = -\dfrac{4}{3}$

$ND' - N'D = 0$

$(1)(3s^2 + 8s + 4) = 0$

$\therefore s = -2, -\frac{2}{3}$

Char. eq.: $s^3 + 4s^2 + 4s + K = 0$

s^3	1	4
s^2	4	K
s^1	$(16-K)/4$	
s^0	K	

for $K = 16$:

$Q_a(s) = 4s^2 + 16 = 4(s^2+4)$

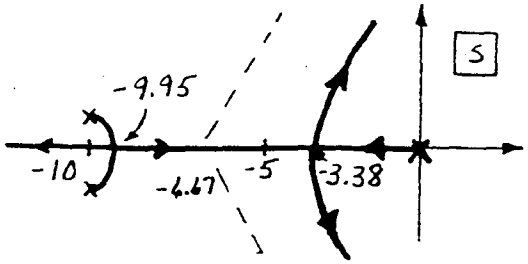
$\therefore s = \pm j2$ at $j\omega$-axis crossings.

(c) $KG(s) = \dfrac{K}{s[s^2+20s+101]} = \dfrac{K}{s^3+20s^2+101s}$

breakaway: $ND' - DN' = 3s^2 + 40s + 101 = 0$

$\therefore s = -3.38, -9.95$

$\sigma_a = \dfrac{-10+j1-10-j1}{3} = -\dfrac{20}{3} = -6.67$



Char. eq: $s^3+20s^2+101s+K=0$

s^3	1	101
s^2	20	K
s^1	$(2020-K)/20$	
s^0	K	

At $K=2020$:

$Q_a(s) = 20s^2+2020$

$=20(s^2+101)$

$\therefore s = \pm j\sqrt{101} = \pm j\,10.05$

at $j\omega$-axis crossing.

(d). $KG(s) = \dfrac{K}{s^3+10s^2+50s}$

Breakaway: $ND'-DN' = (1)(3s^2+20s+50)=0$

both roots complex, $\therefore$ no breakaway points

$\sigma_a = \dfrac{-5+j5-5-j5}{3} = -\dfrac{10}{3}$

Char. eq.: $s^3+10s^2+50s+K=0$

s^3	1	50
s^2	10	K
s^1	$(500-K)/10$	
s^0	K	

At $K=500$:

$Q_a = 10s^2+500$

$=10(s+50)=0$

$\therefore s = \pm j\,7.07$ at

$j\omega$-axis crossing.

$\theta_d = 180^\circ - 90^\circ - 135^\circ$

$= -45^\circ$