4.5. (a) $T(s) = \dfrac{K/s^2}{1 + KK_v/s + K/s^2} = \dfrac{K}{s^2 + KK_v s + K}$

(b) dc gain $= T(0) = 1$, $\therefore$ $\theta_{ss}(t) = 10^\circ$

(c) Critical damping, $\zeta = 1$

$\omega_n^2 = K$; $\omega_n = \sqrt{K}$

$2\zeta\omega_n = 2\omega_n = 2\sqrt{K} = KK_v$; $\therefore$ $K_v = 2(K)^{\frac{1}{2}}$

(d) $\tau = 1.5\,s = \dfrac{1}{\zeta\omega_n} = \dfrac{1}{\sqrt{K}}$, $\therefore$ $\sqrt{K} = \dfrac{1}{1.5} = 0.667$

$\therefore$ $K = 0.444$; $K_v = \dfrac{2}{0.667} = 3$

(f) $T(s) = \dfrac{K}{s^2 + K}$, $\therefore$ $\zeta = 0$

Response is an undamped sinusoid, with
$\omega = \sqrt{K} = 0.667$, $\therefore$ $T = \dfrac{2\pi}{\omega} = 9.42\,s$

5.4. (a) unstable, pole at $s=1$ (b) stable

(c) stable (d) unstable, pole at $s=1$

(e) unstable, poles at $s = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ (f) marginally stable, $s = \pm j\sqrt{3}$

5.6. $T(s) = \dfrac{\frac{K}{s+\alpha}}{1 + \frac{K\beta}{s+\alpha}} = \dfrac{K}{s + \alpha + K\beta}$

(a) $S_K^T = \dfrac{\partial T}{\partial K} \cdot \dfrac{K}{T} = \dfrac{s + \alpha + K\beta - K\beta}{(s + \alpha + K\beta)^2} \cdot \dfrac{K}{K/(s+\alpha+K\beta)} = \dfrac{s+\alpha}{s+\alpha+K\beta}$

$S_K^T\Big|_{s=j\omega} = \dfrac{\alpha + j\omega}{\alpha + K\beta + j\omega} = \dfrac{2 + j\omega}{32 + j\omega}$

(b) $S_\alpha^T(j\omega) = \dfrac{\partial T}{\partial \alpha} \cdot \dfrac{\alpha}{T} = \dfrac{-K}{(s+\alpha+K\beta)^2} \cdot \dfrac{\alpha}{K/(s+\alpha+K\beta)} = \dfrac{-\alpha}{s + \alpha + K\beta}$

$\therefore$ $S_\alpha^T(j\omega) = \dfrac{-2}{32 + j\omega}$

(c) $S_\beta^T = \dfrac{\partial T}{\partial \beta} \cdot \dfrac{\beta}{T} = \dfrac{-K(K)}{(s+\alpha+K\beta)^2} \cdot \dfrac{\beta}{K/(s+\alpha+K\beta)} = \dfrac{-\beta K}{s + \alpha + K\beta}$

$\therefore$ $S_\beta^T(j\omega) = \dfrac{-30}{32 + j\omega}$

5.11. (a) $\frac{C(s)}{D(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)H(s)} = \frac{1}{1+\frac{10K}{5s+1}(1)} = \frac{5s+1}{5s+(1+10K)}$

$C_d(s) = \frac{5s+1}{5s+(1+10K)} \cdot \frac{5}{s}$

$\therefore C_{dss} = \lim_{s\to 0} s\,C_d(s) = \frac{5}{1+10K} = 5(0.01)$

$\therefore 1+10K = 100 \Rightarrow K = \underline{9.9}$

(b) $\frac{C(s)}{R(s)} = T(s) = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)H} = \frac{\frac{10K}{5s+1}}{1+\frac{10K}{5s+1}} = \frac{10K}{5s+10K+1}$

$C_{ss} = \lim_{s\to 0} s\,C(s) = \lim_{s\to 0}\left[\frac{10Ks}{5s+10K+1}\cdot\frac{10}{s}\right] = \left(\frac{10K}{1+10K}\right)10$

$= \frac{99}{100}(10) = 9.9$

$e_{ss} = r_{ss} - C_{ss} = 10 - 9.9 = 0.1$

% error $= \frac{0.1}{10}(100) = \underline{1\%}$