Problem 7.27 Determine the equivalent impedance:

(a) $\mathbf{Z}_1$ at 1000 Hz (Fig. P7-27)(a))

(b) $\mathbf{Z}_2$ at 500 Hz (Fig. P7-27)(b))

(c) $\mathbf{Z}_3$ at $\omega = 10^6$ rad/s (Fig. P7-27)(c))

(d) $\mathbf{Z}_4$ at $\omega = 10^5$ rad/s (Fig. P7-27)(d))

(e) $\mathbf{Z}_5$ at $\omega = 2000$ rad/s (Fig. P7-27)(e))

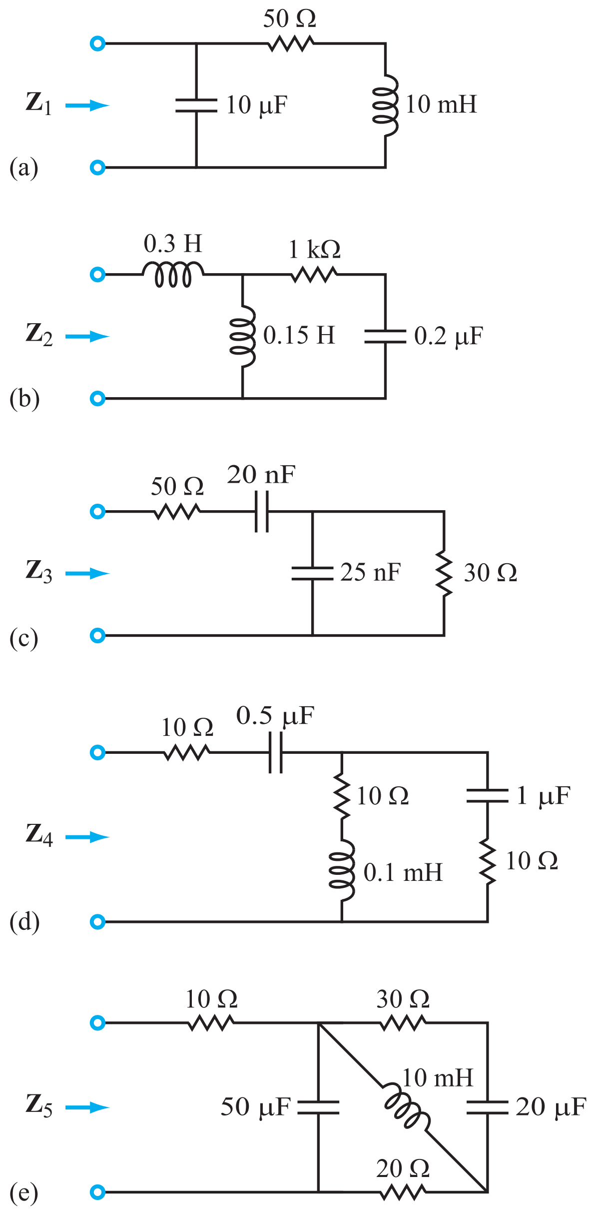
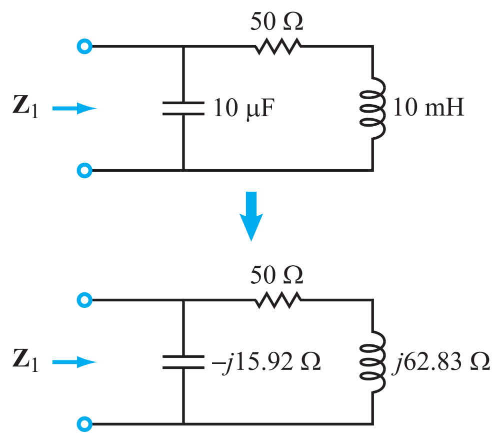


Figure P7.27: Circuit for Problem 7.27.

Solution:

(a)

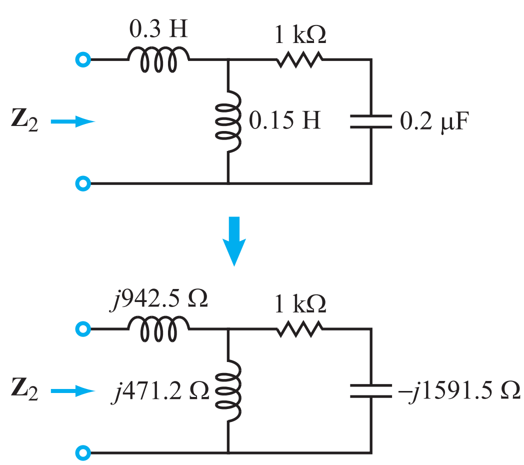


$$\mathbf{Z}_\mathrm{C} = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 10^3 \times 10 \times 10^{-6}} = -j15.92\ \Omega$$

$$\mathbf{Z}_\mathrm{L} = j\omega L = j2\pi \times 10^3 \times 10^{-2} = j62.83\ \Omega$$

$$\begin{aligned}
\mathbf{Z}_1 &= (50 + j62.83) \parallel (-j15.92) \\
&= \frac{(50 + j62.83)(-j15.92)}{50 + j62.83 - j15.92} \\
&= \frac{1000 - j796}{50 + j46.91} \cdot \left(\frac{50 - j46.91}{50 - j46.91}\right) \\
&= \frac{12660 - j86710}{4701} = (2.7 - j18.5)\ \Omega.
\end{aligned}$$

(b)



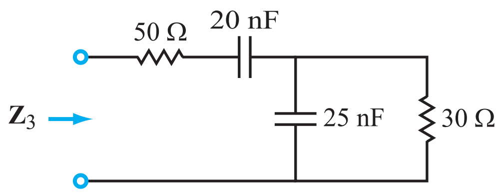
$$\mathbf{Z}_{L_1} = j\omega L_1 = j2\pi \times 500 \times 0.3 = j942.5\ \Omega$$

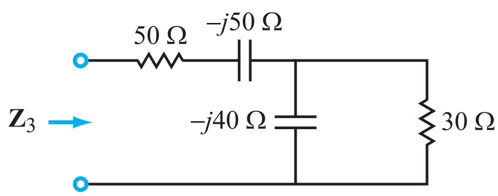
$$\mathbf{Z}_{L_2} = j\omega L_2 = j471.2\ \Omega$$

$$\mathbf{Z}_\mathrm{C} = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 500 \times 0.2 \times 10^{-6}} = -j1591.5\ \Omega$$

$$\begin{aligned}
\mathbf{Z}_2 &= j942.5 + j471.2 \parallel (1000 - j1591.5) \\
&= j942.5 + \frac{j471.2(1000 - j1591.5)}{1000 - j1591.5 + j471.2} = (98.5 + j1524.0)\ \Omega
\end{aligned}$$

(c)



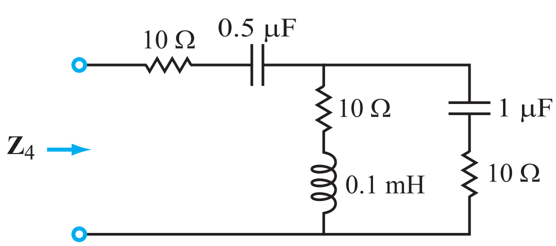


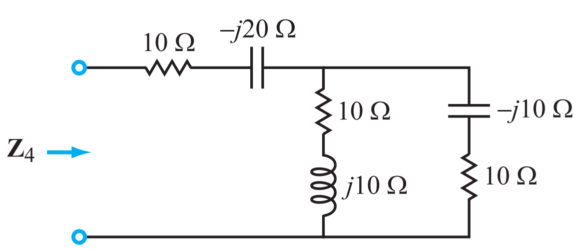
$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{10^6 \times 20 \times 10^{-9}} = -j50\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{10^6 \times 25 \times 10^{-9}} = -j40\ \Omega$$

$$\mathbf{Z}_3 = 50 - j50 + \frac{30 \times (-j40)}{30 - j40} = (69.2 - j64.4)\ \Omega.$$

(d)





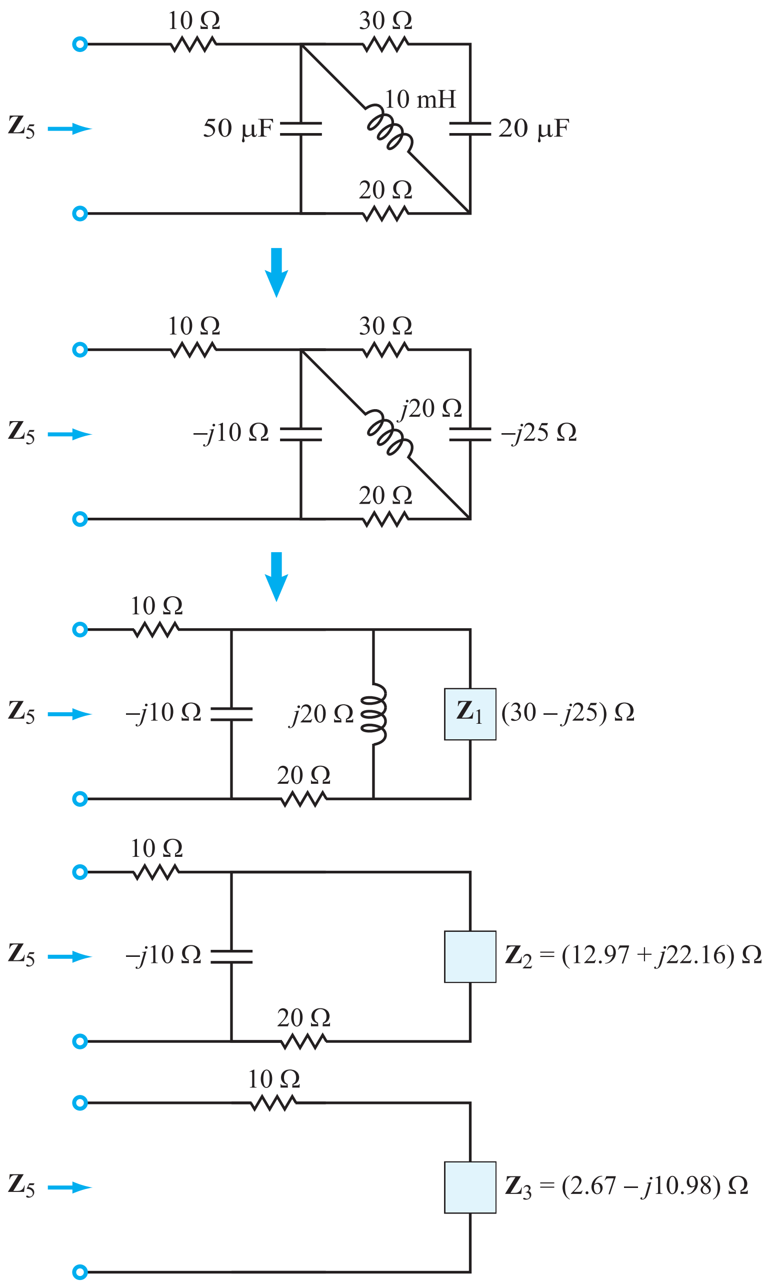
$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{10^5 \times 0.5 \times 10^{-6}} = -j20\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = -j10\ \Omega$$

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j \times 10^5 \times 10^{-4} = j10\ \Omega$$

$$\mathbf{Z}_4 = (10 - j20) + \frac{(10 + j10)(10 - j10)}{10 + j10 + 10 - j10} = (10 - j20) + 10 = (20 - j20)\ \Omega.$$

(e)



$$\mathbf{Z}_{C_1} = \frac{-j}{\omega C_1} = \frac{-j}{2000 \times 50 \times 10^{-6}} = -j10\ \Omega$$

$$\mathbf{Z}_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{2000 \times 20 \times 10^{-6}} = -j25\ \Omega$$

$$\mathbf{Z}_{\mathrm{L}} = j\omega L = j2000 \times 10 \times 10^{-3} = j20\ \Omega$$

$$\begin{aligned}\mathbf{Z}_2 &= \mathbf{Z}_1 \parallel (j20) \\ &= \frac{(30 - j25)(j20)}{30 - j25 + j20} \\ &= \frac{500 + j600}{30 - j5} \cdot \frac{(30 + j5)}{(30 + j5)}\end{aligned}$$

$$
\begin{aligned}
&= \frac{12000 + j20500}{925} = (12.97 + j22.16)\ \Omega \\
\mathbf{Z}_3 &= (20 + \mathbf{Z}_2) \parallel (-j10) \\
&= \frac{(32.97 + j22.16)(-j10)}{32.97 + j22.16 - j10} \\
&= \frac{221.6 - j329.7}{32.97 + j12.16} \cdot \frac{(32.97 - j12.16)}{(32.97 - j12.16)} \\
&= (2.67 - j10.98)\ \Omega \\
\mathbf{Z}_5 &= 10 + \mathbf{Z}_3 = (12.67 - j10.98)\ \Omega.
\end{aligned}
$$

Problem 7.31 In response to an input signal voltage $v_s(t) = 24\cos 2000\pi t$, the input current in the circuit of Fig. P7.31 was measured as $i(t) = 6\cos(2000\pi t - 60^\circ)$ mA. Determine the equivalent input impedance $\mathbf{Z}$ of the circuit.

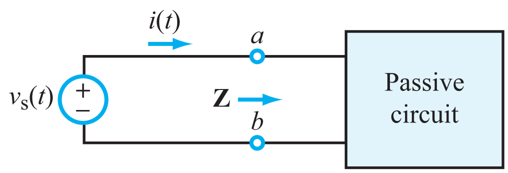


Figure P7.31: Configuration for Problem 7.31.

Solution:

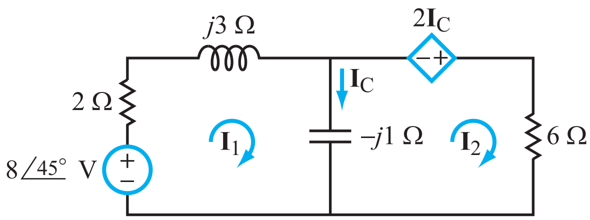
$$\mathbf{V}_s = 24 \text{ V},$$

$$\mathbf{I} = 6e^{-j60^\circ} \qquad \text{(mA)}.$$

$$\begin{aligned}
\mathbf{Z} &= \frac{\mathbf{V}}{\mathbf{I}} \\
&= \frac{24}{6e^{-j60^\circ} \times 10^{-3}} \\
&= 4e^{j60^\circ} \text{ k}\Omega \\
&= (4\cos 60^\circ + j4\sin 60^\circ) \text{ k}\Omega = (2 + j3.46) \text{ k}\Omega.
\end{aligned}$$

Problem 7.52 Apply mesh analysis to determine $\mathbf{I}_C$ in the circuit of Fig. P7.51.

Solution:



$$-8e^{j45^\circ} + 2\mathbf{I}_1 + j3\mathbf{I}_1 - j1(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$-j1(\mathbf{I}_2 - \mathbf{I}_1) - 2\mathbf{I}_C + 6\mathbf{I}_2 = 0$$

Also,

$$\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2.$$

Solution gives

$$\mathbf{I}_1 = \frac{64 - j8}{19 + j16}\, e^{j45^\circ} \quad \text{(A)} \qquad \mathbf{I}_2 = \frac{16 - j8}{19 + j16}\, e^{j45^\circ} \quad \text{(A)}$$

$$\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2 = \frac{48e^{j45^\circ}}{19 + j16} = 1.93e^{j4.9^\circ}.$$

Problem 7.53 Apply mesh analysis to determine $i_L(t)$ in the circuit of Fig. P7.53.

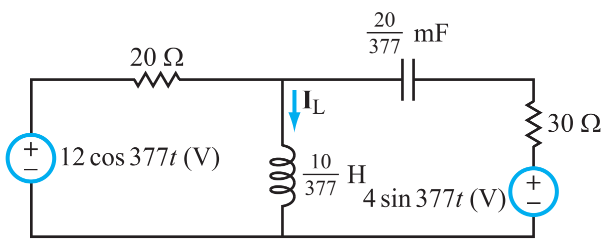
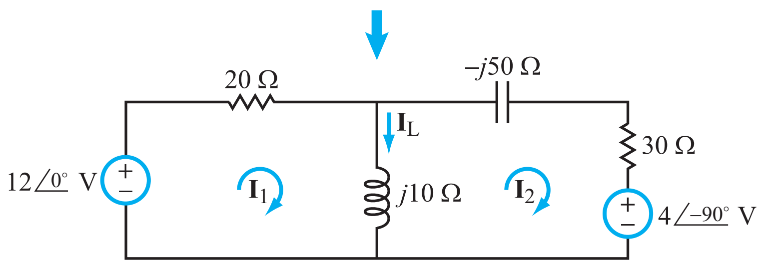


Figure P7.53: Circuit for Problem 7.53.

Solution:



$$12\cos 377t \Longrightarrow 12\angle 0^\circ$$

$$4\sin 377t \Longrightarrow 4\angle -90^\circ$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{377 \times \frac{50}{377}} = -j50\ \Omega$$

$$\mathbf{Z}_L = j\omega L = j377 \times \frac{10}{377} = j10\ \Omega$$

Mesh 1:

$$-12 + 20\mathbf{I}_1 + j10(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

Mesh 2:

$$j10(\mathbf{I}_2 - \mathbf{I}_1) - j50\mathbf{I}_2 + 30\mathbf{I}_2 + 4e^{-j90^\circ} = 0$$

Solution leads to

$$\mathbf{I}_1 = 0.48e^{-j31.87^\circ} \quad \text{(A)}, \qquad \mathbf{I}_2 = 0.17e^{j125.75^\circ} \quad \text{(A)},$$

$$\begin{aligned}
\mathbf{I}_L &= \mathbf{I}_1 - \mathbf{I}_2 \\
&= 0.64e^{-j37.66^\circ} \\
i_L(t) &= \mathfrak{Re}[\mathbf{I}_L e^{j377t}] \\
&= 0.64\cos(377t - 37.66^\circ) \qquad \text{(A)}.
\end{aligned}$$

Problem 7.55 Apply the by-inspection method to develop a mesh-current matrix equation for the circuit in Fig. P7.55, and then use MATLAB® software to solve for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$.

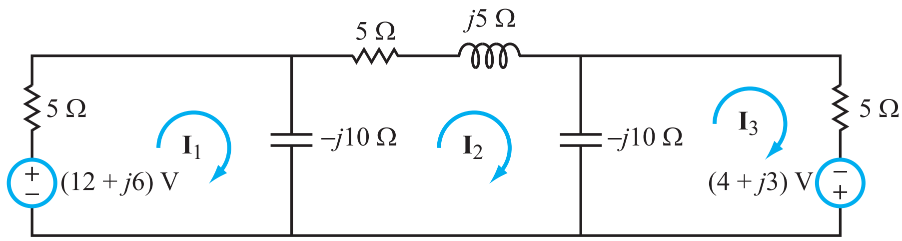


Figure P7.55: Circuit for Problem 7.55.

Solution:

$$\begin{bmatrix} (5-j10) & j10 & 0 \\ j10 & (5+j5-j10) & j10 \\ 0 & j10 & (5-j10) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 12+j6 \\ 0 \\ 4+j3 \end{bmatrix}$$

MATLAB® software solution gives

$$\mathbf{I}_1 = (0.38 + j0.42) \text{ A}$$
$$\mathbf{I}_2 = (0.77 - j0.59) \text{ A}$$
$$\mathbf{I}_3 = (0.30 - j0.34) \text{ A}$$

Problem 7.56 Use any analysis technique of your choice to determine $i_C(t)$ in the circuit of Fig. P7-56.

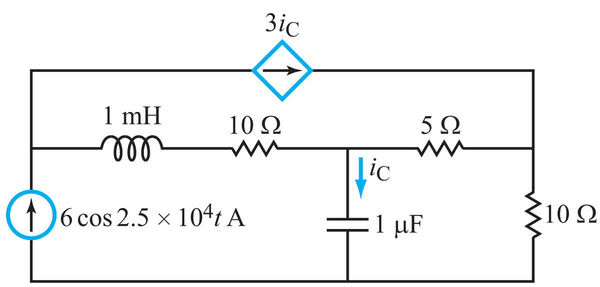
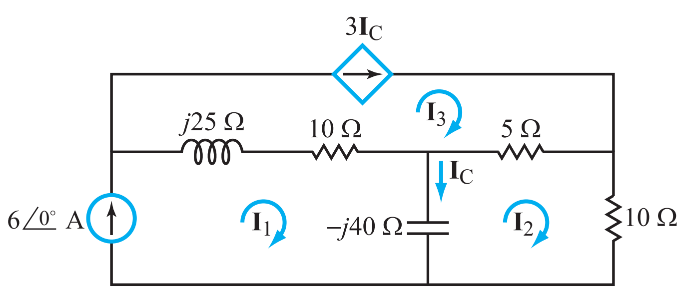


Figure P7.56: Circuit for Problem 7.56.

Solution:



$$\mathbf{Z}_L = j\omega L = j2.5\times10^4\times10^{-3} = j25\ \Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{2.5\times10^4\times10^{-6}} = -j40\ \Omega$$

The mesh-current method gives:

Mesh 1: $\mathbf{I}_1 = 6$ A
Mesh 2: $-j40(\mathbf{I}_2-\mathbf{I}_1)+5(\mathbf{I}_2-\mathbf{I}_3)+10\mathbf{I}_2 = 0$
Mesh 3: $\mathbf{I}_3 = 3\mathbf{I}_C$
Auxiliary: $\mathbf{I}_C = \mathbf{I}_1-\mathbf{I}_2$.

Simultaneous solution leads to

$$\mathbf{I}_1 = 6\text{ A},\qquad \mathbf{I}_2 = (4.92-j1.44)\text{ A}.$$

$$\mathbf{I}_C = \mathbf{I}_1-\mathbf{I}_2 = 1.8e^{j53.13^\circ}\text{ A}.$$

$$i_C(t) = \mathfrak{Re}[\mathbf{I}_C e^{j\omega t}] = 1.8\cos(2.5\times10^4 t+53.13^\circ)\qquad\text{(A)}.$$

Problem 7.57 Determine $i_x(t)$ in the circuit of Fig. P7.57, given that $v_s(t) = 6\cos 5\times10^5 t$ V.

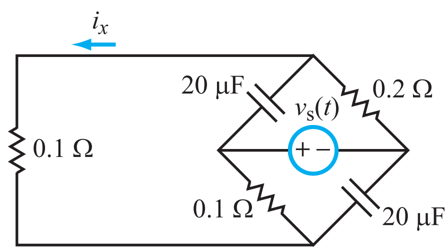
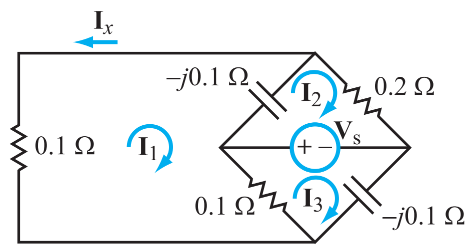


Figure P7.57: Circuit for Problem 7.57.

Solution:



At $\omega = 5\times10^5$ rad/s,

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{5\times10^5\times20\times10^{-6}} = -j0.1\ \Omega$$

$$\mathbf{V}_s = 6\angle 0^\circ\ \text{V}$$

Application of the mesh current by inspection method gives:

$$\begin{bmatrix} (0.1+0.1-j0.1) & j0.1 & -0.1 \\ j0.1 & (0.2-j0.1) & 0 \\ -0.1 & 0 & (0.1-j0.1) \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$$

Solution of matrix equation gives

$$\mathbf{I}_1 = (6.79 - j23.77)\ \text{A}$$

$$\mathbf{I}_2 = (15.85 - j4.53)\ \text{A}$$

$$\mathbf{I}_3 = -(14.72 + j38.49)\ \text{A}$$

$$\mathbf{I}_x = -\mathbf{I}_1 = (6.79 - j23.77)\ \text{A} = 24.72e^{-j74.06^\circ}\ \text{A}$$

$$i_x(t) = 24.72\cos(5\times10^5 t - 74.06^\circ)\ \text{A}.$$